Chapter 9

Exercise 9A

No working is needed for questions 1–5. Refer to the answers in Sadler.

6. (a) $r = \sqrt{3^2 + 3^2}$
$= 3\sqrt{2}$
θ is in quadrant I.
$\tan\theta = \frac{3}{3}$
$\theta = \frac{\pi}{4}$
Polar coordinates are $(3\sqrt{2}, \frac{\pi}{4})$.

(b) $r = \sqrt{1^2 + (\sqrt{3})^2}$
$= 2$
θ is in quadrant I.
$\tan\theta = \frac{\sqrt{3}}{1}$
$\theta = \frac{\pi}{3}$
Polar coordinates are $(2, \frac{\pi}{3})$.

(c) $r = \sqrt{(-2\sqrt{3})^2 + 2^2}$
$= 4$
θ is in quadrant II.
$\tan\theta = \frac{2}{-2\sqrt{3}}$
$= -\frac{1}{\sqrt{3}}$
$\theta = \frac{5\pi}{6}$
Polar coordinates are $(4, \frac{5\pi}{6})$.

(d) $r = \sqrt{(-2\sqrt{3})^2 + (-2)^2}$
$= 4$
θ is in quadrant III.
$\tan\theta = \frac{-2}{-2\sqrt{3}}$
$= \frac{1}{\sqrt{3}}$
$\theta = \frac{7\pi}{6}$
Polar coordinates are $(4, \frac{7\pi}{6})$.

(e) $r = 5$, $\theta = \frac{3\pi}{2}$
Polar coordinates are $(5, \frac{3\pi}{2})$.

(f) $r = \sqrt{7^2 + (-7)^2}$
$= 7\sqrt{2}$
θ is in quadrant IV.
$\tan\theta = \frac{-7}{7}$
$= -1$
$\theta = \frac{7\pi}{4}$
Polar coordinates are $(7\sqrt{2}, \frac{7\pi}{4})$.

(g) $r = 1$, $\theta = \pi$
Polar coordinates are $(1, \pi)$.

(h) $r = \sqrt{(-5)^2 + (-5\sqrt{3})^2}$
$= 10$
θ is in quadrant III.
$\tan\theta = \frac{-5\sqrt{3}}{-5}$
$= \sqrt{3}$
$\theta = \frac{4\pi}{3}$
Polar coordinates are $(10, \frac{4\pi}{3})$.

7. (a) $x = 4\cos 30^\circ$
$= 4 \times \frac{\sqrt{3}}{2}$
$= 2\sqrt{3}$
$y = 4\sin 30^\circ$
$= 4 \times \frac{1}{2}$
$= 2$
Cartesian coordinates are $(2\sqrt{3}, 2)$.

(b) $x = 10\cos 135^\circ$
$= 10 \times -\frac{\sqrt{2}}{2}$
$= -5\sqrt{2}$
$y = 10\sin 135^\circ$
$= 10 \times \frac{\sqrt{2}}{2}$
$= 5\sqrt{2}$
Cartesian coordinates are $(-5\sqrt{2}, 5\sqrt{2})$.

(c) $x = 3\cos(-90^\circ)$
$= 0$
$y = 3\sin(-90^\circ)$
$= -3$
Cartesian coordinates are $(0, -3)$.

(d) $x = 7\sqrt{2}\cos(-135^\circ)$
$= 7\sqrt{2} \times -\frac{\sqrt{2}}{2}$
$= -7$
$y = 7\sqrt{2}\sin(-135^\circ)$
$= 7\sqrt{2} \times -\frac{\sqrt{2}}{2}$
$= -7$
Cartesian coordinates are $(-7, -7)$.

(e) $x = 3\cos 40^\circ$
$= 2.30$
$y = 3\sin 40^\circ$
$= 1.93$
Cartesian coordinates are $(2.30, 1.93)$.

(f) $x = 5\cos(-50°)$
$= 3.21$
$y = 5\sin(-50°)$
$= -3.83$
Cartesian coordinates are $(3.21, -3.83)$.

(g) $x = 4\cos 170°$
$= -3.94$
$y = 4\sin 170°$
$= 0.69$
Cartesian coordinates are $(-3.94, 0.69)$.

(h) $x = 10\cos(-100°)$
$= -1.74$
$y = 10\sin(-100°)$
$= -9.85$
Cartesian coordinates are $(-1.74, -9.85)$.

Miscellaneous Exercise 9

1.
$$\begin{aligned} 3^{x-1} &= 5 \\ \log 3^{x-1} &= \log 5 \\ (x-1)\log 3 &= \log 5 \\ x - 1 &= \frac{\log 5}{\log 3} \\ x &= \frac{\log 5}{\log 3} + 1 \end{aligned}$$

2.
$$\begin{aligned} 3^x - 1 &= 5 \\ 3^x &= 6 \\ \log 3^x &= \log 6 \\ x\log 3 &= \log 6 \\ x &= \frac{\log 6}{\log 3} \end{aligned}$$

3. (a)
$$\begin{aligned} \log_x 64 &= 3 \\ x^3 &= 64 \\ x &= \sqrt[3]{64} \\ &= 4 \end{aligned}$$

(b)
$$\begin{aligned} \log_x 64 &= 2 \\ x^2 &= 64 \\ x &= \sqrt{64} \\ &= 8 \end{aligned}$$

(c)
$$\begin{aligned} \log_x 64 &= 6 \\ x^6 &= 64 \\ x &= \sqrt[6]{64} \\ &= 2 \end{aligned}$$

(d)
$$\begin{aligned} \log_{10} 100 &= x \\ x &= 2 \end{aligned}$$

(e)
$$\begin{aligned} \log 17 - \log 2 &= \log x \\ \log \frac{17}{2} &= x \\ x &= \frac{17}{2} \end{aligned}$$

(f)
$$\begin{aligned} \log 17 + \log 2 &= \log x \\ \log(17 \times 2) &= \log x \\ x &= 34 \end{aligned}$$

(g)
$$\begin{aligned} \log \sqrt{2} &= x\log 2 \\ \log 2^{\frac{1}{2}} &= x\log 2 \\ \frac{1}{2}\log 2 &= x\log 2 \\ x &= \frac{1}{2} \end{aligned}$$

(h)
$$\begin{aligned} 3\log 2 &= \log x \\ \log 2^3 &= \log x \\ x &= 8 \end{aligned}$$

4. (a)
$$\begin{aligned} \mathrm{f}(-21) &= 1 - \frac{1}{\sqrt{4-(-21)}} \\ &= 1 - \frac{1}{\sqrt{25}} \\ &= 1 - \frac{1}{5} \\ &= \frac{4}{5} \end{aligned}$$

(b)
$$\begin{aligned} \mathrm{f}\left(\mathrm{f}(3)\right) &= \mathrm{f}\left(1 - \frac{1}{\sqrt{4-(3)}}\right) \\ &= \mathrm{f}\left(1 - \frac{1}{\sqrt{1}}\right) \\ &= \mathrm{f}(1-1) \\ &= \mathrm{f}(0) \\ &= 1 - \frac{1}{\sqrt{4-0}} \\ &= 1 - \frac{1}{\sqrt{4}} \\ &= 1 - \frac{1}{2} \\ &= \frac{1}{2} \end{aligned}$$

(c) Domain: the square root may not be negative so

$$4 - x \geq 0$$
$$x \leq 4$$

In addition, the denominator of the fraction may not be zero so

$$\sqrt{4-x} \neq 0$$
$$4 - x \neq 0$$
$$x \neq 4$$

Combining these we obtain a domain $\{x \in \mathbb{R} : x < 4\}$

(d) Range: the fraction can not be zero, neither can it be negative (since both numerator and denominator are positive), so $y < 1$ and the range is $\{y \in \mathbb{R} : y < 1\}$

(e) Domain and range of $\mathrm{f}^{-1}(x)$ are the range and domain respectively of $\mathrm{f}(x)$. Domain: $\{x \in \mathbb{R} : x < 1\}$; Range: $\{y \in \mathbb{R} : y < 4\}$

$$\begin{aligned} y &= 1 - \frac{1}{\sqrt{4-x}} \\ \frac{1}{\sqrt{4-x}} &= 1 - y \\ \sqrt{4-x} &= \frac{1}{1-y} \\ 4 - x &= \left(\frac{1}{1-y}\right)^2 \\ x &= 4 - \left(\frac{1}{1-y}\right)^2 \\ &= 4 - \frac{1}{(1-y)^2} \\ \mathrm{f}^{-1}(x) &= 4 - \frac{1}{(1-x)^2} \end{aligned}$$

5. (a) $\overrightarrow{\mathrm{PQ}} = \overrightarrow{\mathrm{Q}} - \overrightarrow{\mathrm{P}}$
$= (13\mathbf{i} - 2\mathbf{j}) - (-7\mathbf{i} + 13\mathbf{j})$
$= 20\mathbf{i} - 15\mathbf{j}$

$\mathrm{PQ} = \sqrt{20^2 + 15^2}$
$= 25$

PR:PQ$= 3 : 5$ so PR$=\frac{3}{5}$PQ$= 15$
RQ=PQ−PR$= 25 - 15 = 10$units

(b) $\overrightarrow{\mathrm{R}} = \overrightarrow{\mathrm{P}} + \frac{3}{5}\overrightarrow{\mathrm{PQ}}$
$= (-7\mathbf{i} + 13\mathbf{j}) + \frac{3}{5}(20\mathbf{i} - 15\mathbf{j})$
$= (-7\mathbf{i} + 13\mathbf{j}) + (12\mathbf{i} - 9\mathbf{j})$
$= 5\mathbf{i} + 4\mathbf{j}$

(c) OR$= \sqrt{5^2 + 4^2} = \sqrt{41} \approx 6.4$units.

6. First solve $|x + 6| = |2x|$:

$x + 6 = 2x$ or $x + 6 = -2x$

$x = 6$ $3x = -6$

$x = -2$

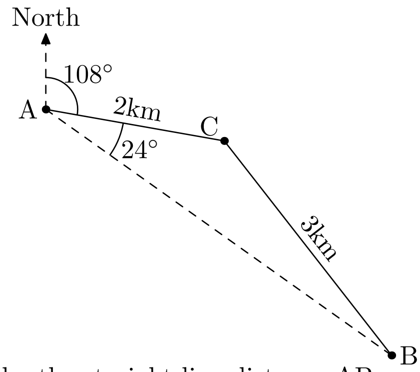
Now test one of the three intervals delimited by these two solutions.

- $x < -2$
  Try a value, say -3:
  Is it true that $|(-3) + 6| \leq |2(-3)|$?
  Yes $(3 \leq 6)$.

Solution set is

$$\{x \in \mathbb{R} : x \leq -2\} \cup \{x \in \mathbb{R} : x \geq 6\}$$

7.



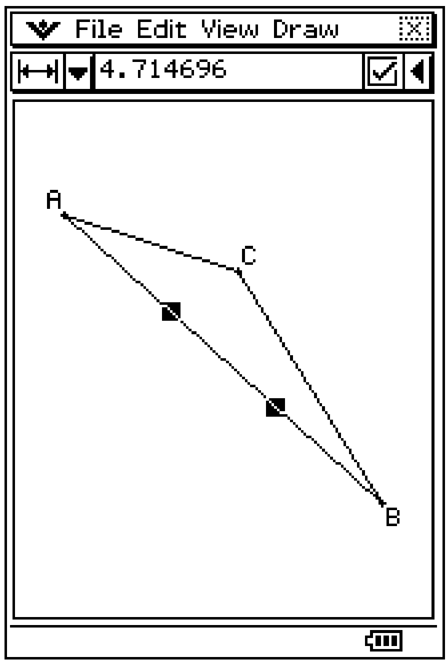
Let x be the straight line distance AB

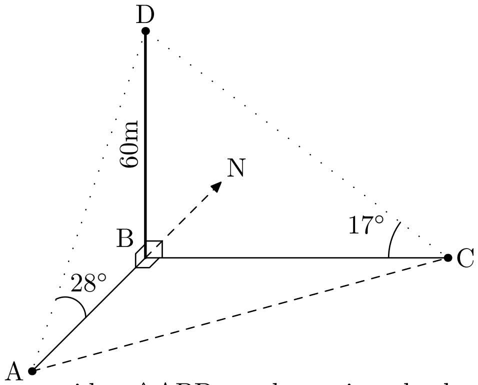
$$\begin{aligned} 3^2 &= x^2 + 2^2 - 2x \times 2\cos 24° \\ x^2 - (4\cos 24°)x + 4 &= 9 \\ x &= 4.715\text{km} \end{aligned}$$

(ignoring the negative root.)

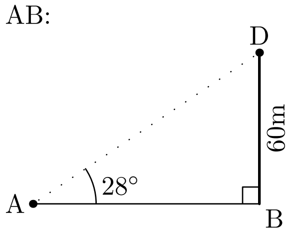
The road route is $2 + 3 - 4.715 = 0.285$km (or about 300m) longer than the straight line distance.

An alternative to solving this algebraically would be to use the geometry app in the ClassPad to construct a scale diagram.



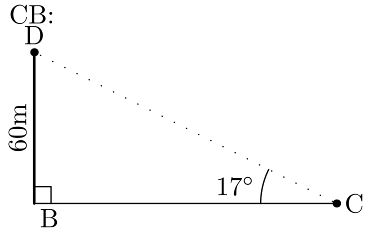
8. 

First consider △ABD to determine the length AB:



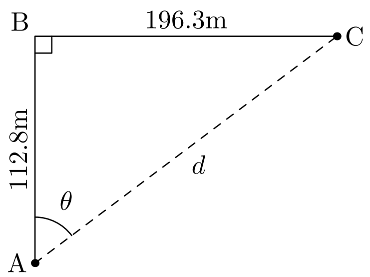
$$\tan 28° = \frac{60}{\text{AB}}$$
$$\text{AB} = \frac{60}{\tan 28°}$$
$$= 112.84\text{m}$$

Next consider △CBD to determine the length CB:



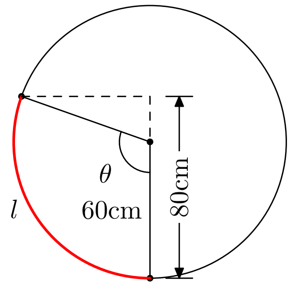
$$\tan 17° = \frac{60}{\text{CB}}$$
$$\text{CB} = \frac{60}{\tan 17°}$$
$$= 196.25\text{m}$$

Finally consider △ABC to determine the length and direction of AC:



$$d = \sqrt{112.84^2 + 196.25^2}$$
$$= 226.38\text{m}$$
$$\tan\theta = \frac{196.25}{112.84}$$
$$\theta = 60.10°$$

C is 226m from A on a bearing of 060°.

9. 

(a) $\cos(\pi - \theta) = \dfrac{80 - 60}{60}$

$$\pi - \theta = \cos^{-1}\frac{20}{60}$$
$$= 1.23$$
$$\theta = \pi - 1.23$$
$$= 1.91$$

(b) $l = r\theta = 60 \times 1.91 \approx 115\text{cm}$

10. $\text{f} \circ \text{g}(x) = \text{f}(2x - 1) = \dfrac{3}{2x - 1}$

$\text{g} \circ \text{f}(x) = \text{g}\left(\dfrac{3}{x}\right) = 2(\dfrac{3}{x}) - 1 = \dfrac{6}{x} - 1$

$\text{f} \circ \text{g}(x)$ has domain determined by $2x - 1 \neq 0$ so the domain is $\{x \in \mathbb{R} : x \neq 0.5\}$
The range of $\text{f} \circ \text{g}(x)$ is $\{y \in \mathbb{R} : y \neq 0\}$.

$\text{g} \circ \text{f}(x)$ has domain $\{x \in \mathbb{R} : x \neq 0\}$.
The range of $\text{g} \circ \text{f}(x)$ is determined by $\frac{6}{x} \neq 0$ so $\frac{6}{x} - 1 \neq -1$ and the range is $\{y \in \mathbb{R} : y \neq -1\}$.

11. Let the original quantity be q. The amount remaining after t years is $q(0.95)^t$.

$$q(0.95)^t = 0.2q$$
$$0.95^t = 0.2$$
$$\log(0.95)^t = \log 0.2$$
$$t \log 0.95 = \log 0.2$$
$$t = \frac{\log 0.2}{\log 0.95}$$
$$= 31.4$$

The company can expect the field to remain profitable for 31 years. It will become unprofitable part-way through the 32nd year.

12. (a) $\log_c 5 = \log_c \dfrac{10}{2}$

$$= \log_c 10 - \log_c 2$$
$$= q - p$$

(b) $\log_c 40 = \log_c(2^2 \times 10)$

$$= 2\log_c 2 + \log_c 10$$
$$= 2p + q$$

(c) $\log_c 200 = \log_c(2 \times 10^2)$

$$= \log_c 2 + 2\log_c 10$$
$$= p + 2q$$

(d) $\log_c(8c) = \log_c(2^3 \times c)$
$= 3\log_c 2 + \log_c c$
$= 3p + 1$

(e)
$$2^{(\log_2 10)} = 10$$
$$log_c 2^{(\log_2 10)} = \log_c 10$$
$$\log_2 10 \log_c 2 = \log_c 10$$
$$\log_2 10 = \frac{\log_c 10}{\log_c 2}$$
$$= \frac{q}{p}$$

(f)
$$10^{(\log 2)} = 2$$
$$log_c 10^{(\log 2)} = \log_c 2$$
$$\log 2 \log_c 10 = \log_c 2$$
$$\log 2 = \frac{\log_c 2}{\log_c 10}$$
$$= \frac{p}{q}$$

13. (a) $_{\mathrm{C}}\mathbf{r}_{\mathrm{A}} = {_{\mathrm{C}}\mathbf{r}_{\mathrm{B}}} + {_{\mathrm{B}}\mathbf{r}_{\mathrm{A}}}$
$= (6\mathbf{i} - \mathbf{j}) + (4\mathbf{i} + 5\mathbf{j})$
$= 10\mathbf{i} + 4\mathbf{j}$
$\mathrm{AC} = \sqrt{10^2 + 4^2}$
$= \sqrt{116}$
$= 2\sqrt{29}$

(b) $\mathbf{r}_{\mathrm{C}} = {_{\mathrm{C}}\mathbf{r}_{\mathrm{A}}} + \mathbf{r}_{\mathrm{A}}$
$= (10\mathbf{i} + 4\mathbf{j}) + (-4\mathbf{i} + 6\mathbf{j})$
$= 6\mathbf{i} + 10\mathbf{j}$

(c) $\mathbf{r}_{\mathrm{A}} + 0.5\overrightarrow{\mathrm{AC}} = (-4\mathbf{i} + 6\mathbf{j}) + 0.5(10\mathbf{i} + 4\mathbf{j})$
$= (-4\mathbf{i} + 6\mathbf{j}) + (5\mathbf{i} + 2\mathbf{j})$
$= \mathbf{i} + 8\mathbf{j}$

14. (a) Points P_1 and P_2 lie on the y-axis, so $x = 0$ for both. For P_1, $y = |x - a| = |0 - a| = a$ so the coordinates of P_1 are $(0, a)$.
For P_2, $y = |0.5x - b| = |0.5(0) - b| = b$ so the coordinates of P_2 are $(0, b)$.

(b) Since P_1 is above P_2 we can conclude $a > b$.

(c) For P_4, $|x - a| = 0$ so $x = a$ and the coordinates are $(a, 0)$.
For P_6, $|0.5x - b| = 0$ so $x = 2b$ and the coordinates are $(2b, 0)$.

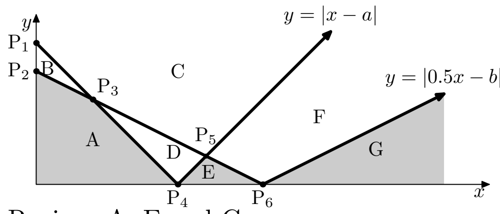
(d) At P_3,

$$-(x - a) = -(0.5x - b)$$
$$x - a = 0.5x - b$$
$$0.5x = a - b$$
$$x = 2a - 2b$$
$$y = -(x - a)$$
$$= -(2a - 2b - a)$$
$$= -(a - 2b)$$
$$= 2b - a$$

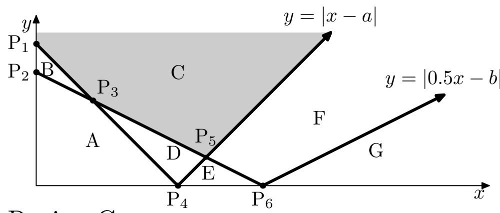
so the coordinates of P_3 are $(2a - 2b, 2b - a)$
At P_5,

$$x - a = -(0.5x - b)$$
$$x - a = -0.5x + b$$
$$1.5x = a + b$$
$$x = \frac{2a + 2b}{3}$$
$$y = x - a$$
$$= \frac{2a + 2b}{3} - a$$
$$= \frac{2a + 2b - 3a}{3}$$
$$= \frac{2b - a}{3}$$

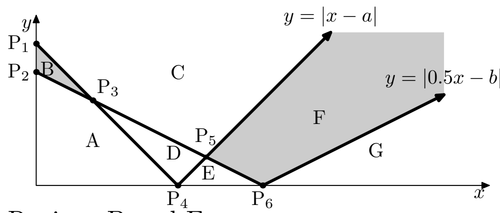
so the coordinates of P_5 are $\left(\frac{2a+2b}{3}, \frac{2b-a}{3}\right)$

(e)


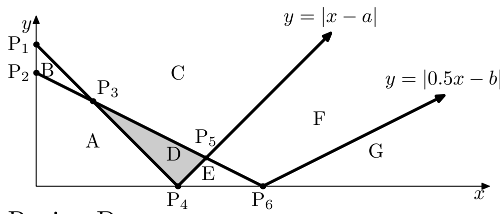
Regions A, E and G.

(f)


Region C.

(g)


Regions B and F.

(h)


Region D.

15. (a) $\overrightarrow{\mathrm{AC}} = 2\overrightarrow{\mathrm{AB}} = 2\mathbf{b}$

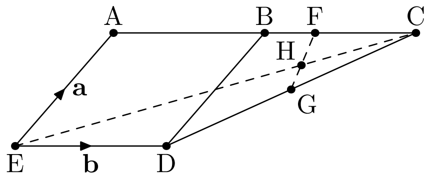
(b) $\overrightarrow{\mathrm{AF}} = \overrightarrow{\mathrm{AB}} + \overrightarrow{\mathrm{BF}} = \mathbf{b} + \frac{1}{3}\mathbf{b} = \frac{4}{3}\mathbf{b}$

(c) $\overrightarrow{\mathrm{DC}} = \overrightarrow{\mathrm{DB}} + \overrightarrow{\mathrm{BC}} = \mathbf{a} + \mathbf{b}$

(d) $\overrightarrow{\mathrm{EC}} = \overrightarrow{\mathrm{ED}} + \overrightarrow{\mathrm{DC}} = \mathbf{b} + \mathbf{a} + \mathbf{b} = \mathbf{a} + 2\mathbf{b}$

(e) $\overrightarrow{\mathrm{EG}} = \overrightarrow{\mathrm{ED}} + \frac{1}{2}\overrightarrow{\mathrm{DC}} = \mathbf{b} + \frac{1}{2}(\mathbf{a} + \mathbf{b}) = \frac{1}{2}\mathbf{a} + \frac{3}{2}\mathbf{b}$

(f) $\overrightarrow{\mathrm{GF}} = \overrightarrow{\mathrm{GC}} + \overrightarrow{\mathrm{CF}} = \frac{1}{2}(\mathbf{a} + \mathbf{b}) + \frac{2}{3}(-\mathbf{b}) = \frac{1}{2}\mathbf{a} - \frac{1}{6}\mathbf{b}$



$$\overrightarrow{GH} = \overrightarrow{GC} + \overrightarrow{CH}$$

$$h\overrightarrow{GF} = \frac{1}{2}\overrightarrow{DC} - k\overrightarrow{EC}$$

$$h\left(\frac{1}{2}\mathbf{a} - \frac{1}{6}\mathbf{b}\right) = \frac{1}{2}(\mathbf{a} + \mathbf{b}) - k(\mathbf{a} + 2\mathbf{b})$$

$$\frac{3h\mathbf{a} - h\mathbf{b}}{6} = \frac{\mathbf{a} + \mathbf{b} - 2k\mathbf{a} - 4k\mathbf{b}}{2}$$

$$3h\mathbf{a} - h\mathbf{b} = 3\mathbf{a} + 3\mathbf{b} - 6k\mathbf{a} - 12k\mathbf{b}$$

$$3h\mathbf{a} - 3\mathbf{a} + 6k\mathbf{a} = 3\mathbf{b} - 12k\mathbf{b} + h\mathbf{b}$$

$$(3h + 6k - 3)\mathbf{a} = (h - 12k + 3)\mathbf{b}$$

$$3h + 6k - 3 = 0$$

$$h + 2k = 1$$

$$h - 12k + 3 = 0$$

$$h - 12k = -3$$

$$14k = 4$$

$$k = \frac{2}{7}$$

$$h + 2\left(\frac{2}{7}\right) = 1$$

$$h = 1 - \frac{4}{7}$$

$$h = \frac{3}{7}$$