Chapter 6

Exercise 6A

1. $2^{-1} = \frac{1}{2^1} = \frac{1}{2}$
2. $4^8 \div 4^6 = 4^{8-6} = 4^2 = 16$
3. $\left(\frac{3}{2}\right)^2 = \frac{3^2}{2^2} = \frac{9}{4} = 2\frac{1}{4}$
4. $18^0 = 1$
5. $\left(4^{0.5}\right)^6 = 4^{0.5\times6} = 4^3 = 64$
6. $5^6 \times 5^{-8} = 5^{6+-8} = 5^{-2} = \frac{1}{25}$
7. $$\begin{aligned}\frac{3^7 \times 27^2}{3^{14}} &= \frac{3^7 \times \left(3^3\right)^2}{3^{14}} \\ &= \frac{3^7 \times 3^6}{3^{14}} \\ &= \frac{3^{13}}{3^{14}} \\ &= \frac{1}{3}\end{aligned}$$
8. $$\begin{aligned}\frac{5^8 \div 5^4}{125} &= \frac{5^{8-4}}{5^3} \\ &= \frac{5^4}{5^3} \\ &= 5\end{aligned}$$
9. $$\begin{aligned}\frac{7^{10} \div 7^2}{49^2 \times 7^5} &= \frac{7^{10-2}}{\left(7^2\right)^2 \times 7^5} \\ &= \frac{7^8}{7^4 \times 7^5} \\ &= \frac{7^8}{7^9} \\ &= \frac{1}{7}\end{aligned}$$
10. $10\ 000 = 10^4$
11. $0.1 = 10^{-1}$
12. $10^9 \div 10^3 = 10^{9-3} = 10^6$
13. $10^5 \times 1000 = 10^5 \times 10^3 = 10^{5+3} = 10^8$
14. $10^5 \div 1000 = 10^5 \div 10^3 = 10^{5-3} = 10^2$
15. $$\begin{aligned}10^7 \times 10^5 \div 1\ 000\ 000 &= 10^{7+5} \div 10^6 \\ &= 10^{12-6} \\ &= 10^6\end{aligned}$$
16. $\sqrt{10} = 10^{\frac{1}{2}}$
17. $\sqrt[3]{10} = 10^{\frac{1}{3}}$
18. $\sqrt{1000} = \left(10^3\right)^{\frac{1}{2}} = 10^{\frac{3}{2}}$
19. $a^4 \div a^9 = a^{4-9} = a^{-5} \quad \Longrightarrow \quad n = -5$
20. $9 - n = 4 \quad \Longrightarrow \quad n = 5$
21. $n - 9 = 4 \quad \Longrightarrow \quad n = 13$
22. $2 \times 3 = n \quad \Longrightarrow \quad n = 6$
23. $\frac{1}{2} \times n = 5 \quad \Longrightarrow \quad n = 10$
24. $\frac{1}{2} + \frac{1}{3} = n \quad \Longrightarrow \quad n = \frac{5}{6}$
25. $2n + 1 = 7 \quad \Longrightarrow \quad n = 3$
26. $$\begin{aligned}(9-n) - (3+2) &= 3 \\ 9 - n - 5 &= 3 \\ 4 - n &= 3 \quad \Longrightarrow \quad n = 1\end{aligned}$$
27. $5 \times \frac{1}{2} - \frac{1}{2} = n \quad \Longrightarrow \quad n = 2$
28. $a^{4+3} = a^7$
29. $12x^3y^4$
30. $\frac{15a^3b}{10ab^3} = \frac{3}{2}a^2b^{-2} = \frac{3a^2}{2b^2}$
31. $9a^2 \times 8a^6b^3 = 72a^8b^3$
32. $\frac{9a^2}{8a^6b^3} = \frac{9}{8a^4b^3}$
33. $6a^{-1} \times 8b = \frac{48b}{a}$
34. $\frac{6}{3}a^{2--3}b^{-4-1} = 2a^5b^{-5} = \frac{2a^5}{b^5}$
35. $\frac{k^7}{k^3} + \frac{k^3}{k^3} = k^4 + 1$
36. $\frac{p^5}{p^2} - \frac{p^8}{p^2} = p^3 - p^6$
37. $5^{(k+2)-(k-1)} = 5^3 = 125$
38. $\frac{5^n\times5^2-50}{5^n-2} = \frac{25(5^n-2)}{5^n-2} = 25$
39. $\frac{2^h\times2^3+8}{3(2^h+1)} = \frac{8(2^h+1)}{3(2^h+1)} = \frac{8}{3} = 2\frac{2}{3}$

Exercise 6B

1. First determine a by taking the ratio of the numbers in one year to that of the previous year.
$$\frac{16800}{18000} = 0.933$$
$$\frac{15200}{16500} = 0.921$$
$$\frac{14000}{15200} = 0.921$$
$$a \approx 0.92$$
The population each year is about 0.92 that of the previous year.
By 2025 the population is expected to be $18000 \times 0.92^{2025-2002} \approx 2600$

2. First determine a
$$\frac{450}{530} = 0.849$$
$$\frac{385}{450} = 0.856$$
$$\frac{325}{385} = 0.844$$
$$a \approx 0.85$$
Now use this to determine k, the initial population:
$$k \times 0.85^5 = 530$$
$$k = \frac{530}{0.85^5}$$
$$\approx 1200 \text{ frogs}$$

3. (a) From the graph, when $t = 3$, $P \approx 67$.
(b) From the graph, when $t = 8$, $P \approx 29$.
(c) $a^{8-3} = \dfrac{29}{67}$
$$a^5 = 0.43$$
$$a = 0.43^{\frac{1}{5}}$$
$$\approx 0.85$$
$$ka^3 = 67$$
$$k = \frac{67}{0.85^3}$$
$$\approx 110$$
(d) When $t = 0$, $P = k \approx 110$.

4. (a) At $t = 0$ the populations of A and B are 10 000 and 1 000 respectively.
(b) After 3 months ($t = 3$) populations are:
$$\mathrm{P_A} = 10\,000(0.75)^3$$
$$\approx 4\,200$$
$$\mathrm{P_B} = 1\,000(1.09)^3$$
$$\approx 1\,300$$
(c) Calculator shows $xc = 6.1589539$ $yc = 1700.2215$ so we conclude that the populations are equal at $t = 6.2$.

5. (a)

t	0	3	6	10
N	850	640	460	320

Determine an estimate for a:
$$\left(\frac{640}{850}\right)^{\frac{1}{3}} = 0.910$$
$$\left(\frac{460}{640}\right)^{\frac{1}{3}} = 0.896$$
$$\left(\frac{320}{460}\right)^{\frac{1}{4}} = 0.913$$
$$a \approx 0.91$$
An estimate for k can be read directly from the graph—the value of N when t=0: $k \approx 850$.
(b) Solving $(0.91)^t = \frac{1}{4}$ gives $t \approx 14.7$, so we expect releases to cease after 15 weeks.

Exercise 6C

There is no need for worked solutions for questions 1–32 as these are all single step problems.

33. $64 = 8^2$ so $\log_8 64 = 2$

34. $128 = 2^7$ so $\log_2 128 = 7$

35. $10\,000 = 10^4$ so $\log 10\,000 = 4$

36. $243 = 3^5$ so $\log_3 243 = 5$

37. $\frac{1}{2} = 2^{-1}$ so $\log_2\left(\frac{1}{2}\right) = -1$

38. $\frac{1}{16} = 2^{-4}$ so $\log_2\left(\frac{1}{16}\right) = -4$

39. $216 = 6^3$ so $\log_6\left(\frac{1}{216}\right) = -3$

40. $0.125 = \frac{1}{8} = 2^{-3}$ so $\log_2(0.125) = -3$
41. $243 = 3^5 = \left(9^{\frac{1}{2}}\right)^5 = 9^{2.5}$ so $\log_9 243 = 2.5$
42. $0.001 = 10^{-3}$ so $\log(0.001) = -3$
43. $6 = 6^1$ so $\log_6 6 = 1$
44. $1 = 7^0$ so $\log_7 1 = 0$
45. $1 = a^0$ so $\log_a 1 = 0$ (provided $a \neq 0$)
46. $32 = 2^5 = \left(4^{\frac{1}{2}}\right)^5 = 4^{2.5}$ so $\log_4 32 = 2.5$
47. $a = a^1$ so $\log_a a = 1$ $(a \neq 0)$
48. $\log_a\left(a^3\right) = 3$

Questions 49–56 are straightforward calculator exercises so there is no need for worked solutions.

57. (a) Yes: c is negative for all $b < 1$.
    (b) No: there is no value of c such that 10^c is negative so b can never be negative.

Exercise 6D

1. $\log xz$
2. $\log x^2 + \log y = \log x^2 y$
3. $\log x^2 + \log y^3 = \log x^2 y^3$
4. $\log x^2 - \log y = \log \frac{x^2}{y}$
5. $\log ab - \log c = \log \frac{ab}{c}$
6. $\log a^3 + \log b^4 - \log c^2 = \log \frac{a^3 b^4}{c^2}$
7. $$\begin{aligned} 5\log c - 3\log c + \log a &= 2\log c + \log a \\ &= \log c^2 + \log a \\ &= \log ac^2 \end{aligned}$$
8. $\log 100 + \log x = \log 100x$
9. $$\begin{aligned} \log 1000 - (\log x + \log y) &= \log 1000 - \log xy \\ &= \log \frac{1000}{xy} \end{aligned}$$
10. $\log 1000 - \log x + \log y = \log \frac{1000y}{x}$
11. $\log_2\left(\frac{24}{3}\right) = \log_2 8 = 3$
12. $\log_2 \frac{20\times 8}{10} = \log_2 16 = 4$
13. $4 - 1 = 3$
14. $3 + 2 - 4 = 1$
15. $$\begin{aligned} \log_3 45 + \log_3 2^2 - \log_3 20 &= \log_3 \frac{45 \times 4}{20} \\ &= \log_3 9 \\ &= 2 \end{aligned}$$
16. $$\begin{aligned} \log_3 4 - \log_3 36 - 2 &= \log_3 \frac{4}{36} - 2 \\ &= \log_3 \frac{1}{9} - 2 \\ &= -2 - 2 \\ &= -4 \end{aligned}$$
17. $$\begin{aligned} \log 5 - (\log 2 + 2\log 5) &= \log 5 - \log(2 \times 5^2) \\ &= \log 5 - \log 50) \\ &= \log \frac{5}{50} \\ &= \log \frac{1}{10} \\ &= -1 \end{aligned}$$
18. $$\begin{aligned} \log_a b + \log_a(ab)^2 - \log_a b^3 &= \log_a \frac{a^2 b^3}{b^3} \\ &= \log_a a^2 \\ &= 2 \end{aligned}$$
19. $\frac{3\log_a b}{2\log_a b} = \frac{3}{2} = 1.5$
20. $$\begin{aligned} \frac{\log_a \frac{48}{3}}{\log_a 2} &= \frac{\log_a 16}{\log_a 2} \\ &= \frac{\log_a 2^4}{\log_a 2} \\ &= \frac{4\log_a 2}{\log_a 2} \\ &= 4 \end{aligned}$$
21. (a) $\log_a(2 \times 3) = \log_a 2 + \log_a 3 = p + q$
    (b) $\log_a(2 \times 3^2) = \log_a 2 + 2\log_a 3 = p + 2q$
    (c) $\log_a(2^2 \times 3) = 2\log_a 2 + \log_a 3 = 2p + q$
    (d) $\log_a \frac{2}{3} = \log_a 2 - \log_a 3 = p - q$
    (e) $\log_a 3^2 + \log_a a^4 = 2\log_a 3 + 4 = 2q + 4$
    (f) $$\begin{aligned} \log_a \frac{2}{9} &= \log_a 2 - \log_a 3^2 \\ &= \log_a 2 - 2\log_a 3 \\ &= p - 2q \end{aligned}$$
22. (a) $\log_5 7^2 = 2\log_5 7 = 2a$
    (b) $\log_5(2^2 \times 7) = \log_5 7 + 2\log_5 2 = a + 2b$
    (c) $\log_5 \frac{7}{2^2} = \log_5 7 - 2\log_5 2 = a - 2b$
    (d) $\log_5(5^2 \times 2) = \log_5 5^2 + \log_5 2 = 2 + b$

(e) $\log_5(7^2 \times 2 \times 5) = 2\log_5 7 + \log_5 2 + \log_5 5$
$= 2a + b + 1$

(f) $\log_5(7 \times 2^2 \times 5^2) = \log_5 7 + 2\log_5 2 + 2$
$= a + 2b + 2$

23. $y = a^x$

24. $y = 2x$

25. $\log_a y = \log_a x^3$
$y = x^3$

26. $\log_a y^2 = \log_a x^3$
$y^2 = x^3$
$y = x^{\frac{3}{2}}$

27. $\log_a y = \log_a ax$
$y = ax$

28. $\log_a y = \log_a a^2 + \log_a x$
$= \log_a a^2 x$
$y = a^2 x$

29. $\log_a y = \log_a x^{-1}$
$y = x^{-1}$
$y = \dfrac{1}{x}$

30. $\log_a xy = \log_a a^2$
$xy = a^2$
$y = \dfrac{a^2}{x}$

31. (a) $75 - 35\log(0 + 1) = 75$
(b) $75 - 35\log(2 + 1) \approx 58$
(c) $75 - 35\log(4 + 1) \approx 51$
(d) $75 - 35\log(t + 1) = 40$
$35\log(t + 1) = 75 - 40$
$\log(t + 1) = 1$
$t + 1 = 10^1$
$t = 10 - 1$
$= 9$ fortnights

32. (a) $R = \log\left(\frac{1000 I_0}{I_0}\right) = \log 1000 = 3$
(b) $5.4 = \log\left(\dfrac{I}{I_0}\right)$
$\dfrac{I}{I_0} = 10^{5.4}$
$I = 10^{5.4} I_0$
$\approx 250\,000 I_0$
(c) An earthquake measuring 6 has an intensity of $10^6 I_0$ while one measuring 5 has an intensity of $10^5 I_0$, so the former has an intensity **ten times** that of the latter.
(d) $\dfrac{10^{7.7} I_0}{10^{5.9} I_0} = 10^{7.7-5.9}$
$= 10^{1.8}$
≈ 63 times as intense

33. (a) $-\log_{10} 0.0001 = 4.0$
(b) $-\log_{10} 0.000\,031\,6 \approx 4.5$
(c) $-\log_{10} 0.000\,000\,25 \approx 6.6$
(d) $-\log_{10} 0.000\,000\,016 \approx 7.8$
(e) $-\log_{10} 0.000\,000\,042 \approx 7.4$
(f) $10^{-5.25} \approx 0.000\,005\ 62$ mol/L

34. (a) $10\log_{10}\left(\dfrac{I}{I_0}\right) = 40$
$\log_{10}\left(\dfrac{I}{I_0}\right) = 4$
$\dfrac{I}{I_0} = 10^4$
$I = 10\,000 I_0$
(b) $10\log_{10}\left(\dfrac{I}{I_0}\right) = 70$
$\log_{10}\left(\dfrac{I}{I_0}\right) = 7$
$\dfrac{I}{I_0} = 10^7$
$I = 10\,000\,000 I_0$
(c) $10\log_{10}\left(\dfrac{I_1}{I_2}\right) = 90 - 20$
$\log_{10}\left(\dfrac{I_1}{I_2}\right) = \dfrac{70}{10}$
$\dfrac{I_1}{I_2} = 10^7$
The intensity of a 90dB noise level is 10 000 000 times that of a 20dB noise level.

Exercise 6E

1. $$\begin{aligned}\log 3^x &= \log 7\\ x\log 3 &= \log 7\\ x &= \frac{\log 7}{\log 3}\end{aligned}$$

2. $$\begin{aligned}\log 7^x &= \log 1000\\ x\log 7 &= 3\\ x &= \frac{3}{\log 7}\end{aligned}$$

3. $$\begin{aligned}\log 10^x &= \log 27\\ x\log 10 &= \log 27\\ x &= \log 27\\ &= 3\log 3\end{aligned}$$

4. $$\begin{aligned}\log 2^x &= \log 11\\ x\log 2 &= \log 11\\ x &= \frac{\log 11}{\log 2}\end{aligned}$$

5. $$\begin{aligned}\log 3^x &= \log 17\\ x\log 3 &= \log 17\\ x &= \frac{\log 17}{\log 3}\end{aligned}$$

6. $$\begin{aligned}\log 7^x &= \log 80\\ x\log 7 &= \log 80\\ x &= \frac{\log 80}{\log 7}\\ &= \frac{\log\left(10\times 2^3\right)}{\log 7}\\ &= \frac{1+3\log 2}{\log 7}\end{aligned}$$

7. $$\begin{aligned}\log 5^x &= \log 21\\ x\log 5 &= \log 21\\ x &= \frac{\log 21}{\log 5}\end{aligned}$$

8. $$\begin{aligned}\log 10^x &= \log 15\\ x\log 10 &= \log 15\\ x &= \log 15\end{aligned}$$

9. $$\begin{aligned}\log 2^x &= \log 70\\ x\log 2 &= \log 70\\ x &= \frac{\log 70}{\log 2}\\ x &= \frac{\log(7\times 10)}{\log 2}\\ x &= \frac{1+\log 7}{\log 2}\end{aligned}$$

10. $$\begin{aligned}\log 6^{(x+2)} &= \log 17\\ (x+2)\log 6 &= \log 17\\ x+2 &= \frac{\log 17}{\log 6}\\ x &= \frac{\log 17}{\log 6}-2\\ x &= \frac{\log 17}{\log 6}-\frac{2\log 6}{\log 6}\\ &= \frac{\log 17-\log 36}{\log 6}\\ &= \frac{\log\frac{17}{36}}{\log 6}\end{aligned}$$

11. $$\begin{aligned}\log 3^{(x+1)} &= \log 51\\ (x+1)\log 3 &= \log 51\\ x+1 &= \frac{\log 51}{\log 3}\\ x &= \frac{\log(17\times 3)}{\log 3}-1\\ x &= \frac{\log 17+\log 3}{\log 3}-1\\ x &= \frac{\log 17}{\log 3}+1-1\\ x &= \frac{\log 17}{\log 3}\end{aligned}$$

12. $$\begin{aligned}\log 8^{(x-1)} &= \log 7\\ (x-1)\log 8 &= \log 7\\ x-1 &= \frac{\log 7}{\log 8}\\ x &= \frac{\log 7}{\log 8}+1\\ x &= \frac{\log 7}{\log 8}+\frac{\log 8}{\log 8}\\ &= \frac{\log 7+\log 8}{\log 2^3}\\ &= \frac{\log 56}{3\log 2}\end{aligned}$$

13. $$\begin{aligned}\log 5^{(x-1)} &= \log 3^{2x}\\ (x-1)\log 5 &= 2x\log 3\\ x\log 5-\log 5 &= 2x\log 3\\ x\log 5-2x\log 3 &= \log 5\\ x(\log 5-2\log 3) &= \log 5\\ x &= \frac{\log 5}{\log 5-2\log 3}\\ x &= \frac{\log 5}{\log\frac{5}{9}}\end{aligned}$$

14.
$$\begin{aligned}\log 2^{(x+1)} &= \log 3^x\\(x+1)\log 2 &= x\log 3\\x\log 2+\log 2 &= x\log 3\\x\log 2 - x\log 3 &= -\log 2\\x(\log 2-\log 3) &= -\log 2\\x &= -\frac{\log 2}{\log 2-\log 3}\\x &= -\frac{\log 2}{\log\frac{2}{3}} \qquad \text{(see below)}\\&= \frac{\log\frac{1}{2}}{\log\frac{2}{3}}\end{aligned}$$

$$\text{alternatively:}\quad x = \frac{\log 2}{\log 3-\log 2} = \frac{\log\frac{1}{2}}{\log\frac{3}{2}}$$

Although they look different, these two answers are equivalent.

15.
$$\begin{aligned}\log 4^{3x} &= \log 5^{(x+2)}\\3x\log 4 &= (x+2)\log 5\\3x\log 2^2 &= x\log 5+2\log 5\\6x\log 2 &= x\log 5+2\log 5\\x(6\log 2-\log 5) &= 2\log 5\\x &= \frac{2\log 5}{6\log 2-\log 5}\\x &= \frac{2\log 5}{\log\frac{64}{5}}\end{aligned}$$

16.
$$\begin{aligned}\log 3^{(2x+1)} &= \log 2^{(3x-1)}\\(2x+1)\log 3 &= (3x-1)\log 2\\2x\log 3+\log 3 &= 3x\log 2-\log 2\\2x\log 3-3x\log 2 &= -\log 3-\log 2\\x(2\log 3-3\log 2) &= -\log 6\\x(3\log 2-2\log 3) &= \log 6\\x &= \frac{\log 6}{3\log 2-2\log 3}\\x &= \frac{\log 6}{\log\frac{8}{9}}\end{aligned}$$

17.
$$\begin{aligned}5\left(2^x\right) &= 3-2^x\times 2^2\\5\left(2^x\right) &= 3-4\left(2^x\right)\\9\left(2^x\right) &= 3\\\left(2^x\right) &= \frac{1}{3}\\\log\left(2^x\right) &= \log\frac{1}{3}\\x\log 2 &= -\log 3\\x &= -\frac{\log 3}{\log 2}\end{aligned}$$

18.
$$\begin{aligned}5^x+4\left(5^x\times 5^1\right) &= 63\\5^x+20\left(5^x\right) &= 63\\5^x(1+20) &= 63\\5^x &= \frac{63}{21}\\&= 3\\\log 5^x &= \log 3\\x\log 5 &= \log 3\\x &= \frac{\log 3}{\log 5}\end{aligned}$$

19.
$$\begin{aligned}y^2+3y-18 &= 0\\(y+6)(y-3) &= 0\\y &= 3 \qquad \text{(rejecting } y<0)\\2^x &= 3\\x\log 2 &= \log 3\\x &= \frac{\log 3}{\log 2}\end{aligned}$$

20.
$$\begin{aligned}\left(2^x\right)^2-2^3\left(2^x\right)+15 &= 0\\y^2-8y+15 &= 0\\(y-3)(y-5) &= 0\\y &= 3 &&\text{or } y=5\\2^x &= 3 &&\text{or } 2^x=5\\x\log 2 &= \log 3 &&\text{or } x\log 2=\log 5\\x &= \frac{\log 3}{\log 2} &&\text{or } x=\frac{\log 5}{\log 2}\end{aligned}$$

21.
$$\begin{aligned}2^x &= 7\\x\log 2 &= \log 7\\x &= \frac{\log 7}{\log 2}\end{aligned}$$

22. These are single-step problems, following the example of question 21.

23. n must be at least that given by solving

$$\begin{aligned}0.92^n &= 0.2\\n\log 0.92 &= \log 0.2\\n &= \frac{\log 0.2}{\log 0.92}\\&= 19.3\end{aligned}$$

The metal must be passed through the rollers 20 times.

24. (a) $N = 200(2.7)^{0.1\times 3} \approx 270$

(b) $N = 200(2.7)^{0.1\times 5} \approx 330$

(c) $200(2.7)^{0.1t} = 1000$

$$\begin{aligned}2.7^{0.1t} &= 50.1t\log 2.7 \quad = \log 5\\0.1t &= \frac{\log 5}{\log 2.7}\\t &= 10\frac{\log 5}{\log 2.7}\\&\approx 16.2\end{aligned}$$

The population first exceeded 1000 on the 17th day.

25. $$\begin{aligned} 2.8^{20a} &= 51 \\ 20a \log 2.8 &= \log 51 \\ 20a &= \frac{\log 51}{\log 2.8} \\ a &= \frac{\log 51}{20 \log 2.8} \\ &= 0.19 \end{aligned}$$

The percentage blood alcohol level is 0.19%.

26. (a) $N = 100\,000 + 150\,000(1.1)^{-0.8\times 4} \approx 211\,000$

(b) $N = 100\,000 + 150\,000(1.1)^{-0.8\times 8} \approx 182\,000$

$$\begin{aligned} 100\,000 + 150\,000(1.1)^{-0.8t} &= 135\,000 \\ 150\,000(1.1)^{-0.8t} &= 35\,000 \\ 1.1^{-0.8t} &= \frac{35\,000}{150\,000} \\ &= \frac{7}{30} \\ -0.8t \log 1.1 &= \log \frac{7}{30} \\ -0.8t &= \frac{\log \frac{7}{30}}{\log 1.1} \\ t &= -\frac{\log \frac{7}{30}}{0.8 \log 1.1} \\ &\approx 19.1 \end{aligned}$$

Sales fall to 135 000 approximately 19 weeks after the campaign ceases.

27. (a) $P = 10\,000(1.08)^3 = \$12\,597$

(b) $P = 10\,000(1.08)^7 = \$17\,138$

(c) $$\begin{aligned} 10\,000(1.08)^t &= 50\,000 \\ (1.08)^t &= 5 \\ t \log 1.08 &= \log 5 \\ t &= \frac{\log 5}{\log 1.08} \\ &\approx 21 \text{ years} \end{aligned}$$

(d) i. $$\begin{aligned} t &= \frac{\log 5}{\log 1.10} \\ &\approx 17 \text{ years} \end{aligned}$$

ii. After the first 8 years,

$$\begin{aligned} P &= 10\,000(1.14)^8 \\ &= 28\,526 \end{aligned}$$

thereafter: $P = 28\,526(1.10)^{t-8}$ so we need to solve

$$\begin{aligned} 28\,526(1.10)^{t-8} &= 50\,000 \\ 1.10^{t-8} &= \frac{50\,000}{28\,526} \\ (t-8)\log(1.1) &= \log \frac{50\,000}{28\,526} \\ t - 8 &= \frac{\log \frac{50\,000}{28\,526}}{\log 1.1} \\ t &= \frac{\log \frac{50\,000}{28\,526}}{\log 1.1} + 8 \\ &\approx 14 \text{ years} \end{aligned}$$

(e) $$\begin{aligned} (1+r)^5 &= 2 \\ 1 + r &= 2^{\frac{1}{5}} \\ r &= 2^{\frac{1}{5}} - 1 \\ &\approx 0.149 \end{aligned}$$

The required interest rate is 14.9%.

Miscellaneous Exercise 6

1. $$\begin{aligned} \log 2^x &= \log 11 \\ x \log 2 &= \log 11 \\ x &= \frac{\log 11}{\log 2} \end{aligned}$$

2. (a) $$\begin{aligned} \log_a 25 &= \log_a 5^2 \\ &= 2 \log_a 5 \\ &= 2p \end{aligned}$$

(b) $$\begin{aligned} \log_a 20 &= \log_a (5 \times 4) \\ &= \log_a 5 + \log_a 4 \\ &= p + q \end{aligned}$$

(c) $$\begin{aligned} \log_a 80 &= \log_a (5 \times 4^2) \\ &= \log_a 5 + \log_a 4^2 \\ &= \log_a 5 + 2 \log_a 4 \\ &= p + 2q \end{aligned}$$

(d) $$\begin{aligned} \log_a 0.8 &= \log_a \frac{4}{5} \\ &= \log_a 4 - \log_a 5 \\ &= q - p \end{aligned}$$

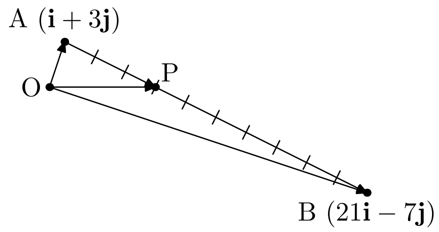
(e) $\log_a 20a^3 = \log_a(5 \times 4 \times a^3)$

$$= \log_a 5 + \log_a 4 + \log_a a^3$$
$$= p + q + 3$$

(f) $\log_5 4 = \dfrac{\log_a 4}{\log_a 5}$

$$= \frac{q}{p}$$

3.



$$\overrightarrow{\text{AP}} = \frac{3}{10}\overrightarrow{\text{AB}}$$
$$\overrightarrow{\text{OP}} - \overrightarrow{\text{OA}} = \frac{3}{10}(\overrightarrow{\text{OB}} - \overrightarrow{\text{OA}})$$
$$\overrightarrow{\text{OP}} = \frac{3}{10}\overrightarrow{\text{OB}} + \frac{7}{10}\overrightarrow{\text{OA}}$$
$$= (6.3\mathbf{i} - 2.1\mathbf{j}) + (0.7\mathbf{i} + 2.1\mathbf{i})$$
$$= 7\mathbf{i}$$

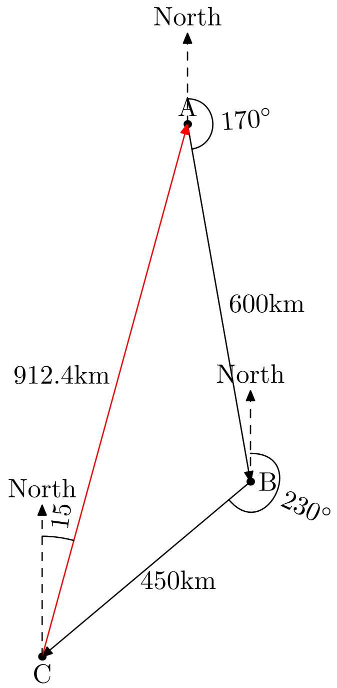
4. $Q = Q_0(1 - 0.12)^t = Q_0(0.88)^t$

For just 5% to remain, $\frac{Q}{Q_0} = 0.05$ so we must solve

$$0.88^t = 0.05$$
$$t \log 0.88 = \log 0.05$$
$$t = \frac{\log 0.05}{\log 0.88}$$
$$\approx 23.4 \text{ minutes}$$

If the 5% is a maximum, we should run the pump for about 24 minutes.

5.

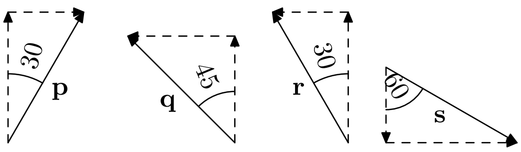


$$\angle\text{ABC} = 360 - 230 - 10$$
$$= 120°$$

$$d = \sqrt{600^2 + 450^2 - 2 \times 600 \times 450 \cos 120°}$$
$$\approx 912\text{km}$$

$$\frac{\sin \angle\text{BAC}}{450} = \frac{\sin 120°}{912.4}$$
$$\angle\text{BAC} = \sin^{-1} \frac{450 \sin 120°}{912.4}$$
$$\approx 25.4°$$
$$\text{bearing of C from A} = 170 + 25.4$$
$$= 195.4°$$
$$\text{bearing of A from C} = 195.4 + 25.4 - 180$$
$$\approx 15°$$

6.



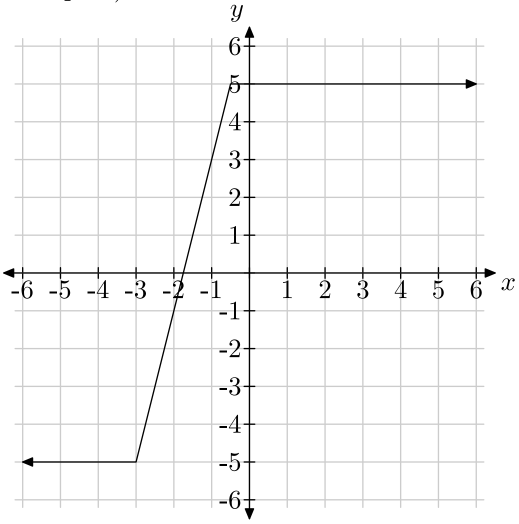
$$\mathbf{p} = 6 \sin 30°\mathbf{i} + 6 \cos 30°\mathbf{j}$$
$$= 3\mathbf{i} + 3\sqrt{3}\mathbf{j}$$

$$\mathbf{q} = -8\sqrt{2} \sin 45°\mathbf{i} + 8\sqrt{2} \cos 45°\mathbf{j}$$
$$= -8\mathbf{i} + 8\mathbf{j}$$

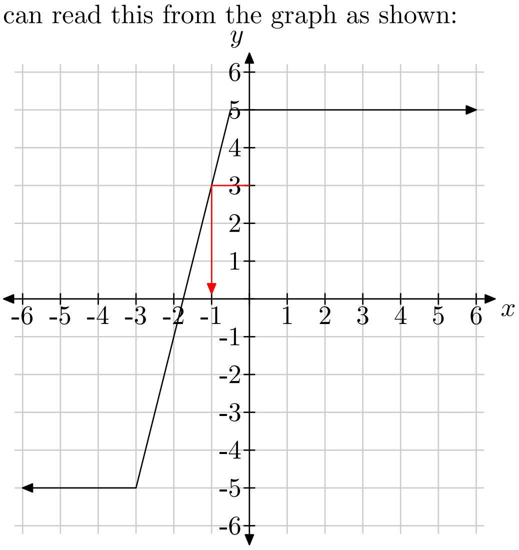
$$\mathbf{r} = -10 \sin 30°\mathbf{i} + 10 \cos 30°\mathbf{j}$$
$$= -5\mathbf{i} + 5\sqrt{3}\mathbf{j}$$

$$\mathbf{s} = 8 \sin 60°\mathbf{i} - 8 \cos 60°\mathbf{j}$$
$$= 4\sqrt{3}\mathbf{i} - 4\mathbf{j}$$

7. Start by drawing a graph of the left hand side: $y = 2|x+3| - |2x+1|$ (If you were to attempt this manually you would need to consider three parts of the domain, just as you would if you were to solve it algebraically. I'll assume that you can draw the graph, at least with the help of your Classpad.)

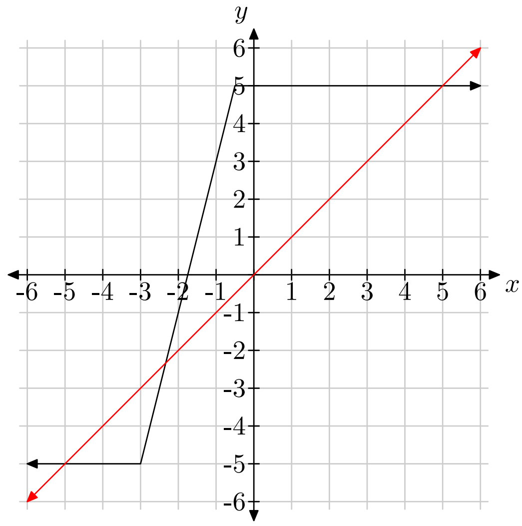


(a) There is just one value of x that results in the left hand side having a value of 3. You can read this from the graph as shown:



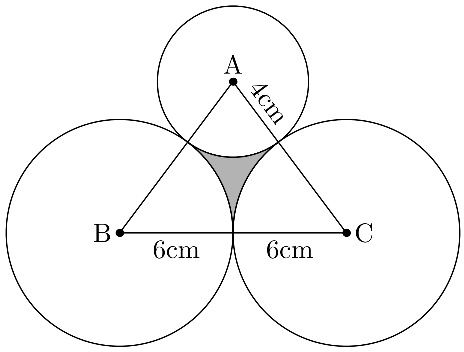
Solution: $x = -1$.

(b) To solve the second equation, superimpose on the graph the line $y = x$ and locate the points where RHS and LHS intersect.



Solution: $x = -5$, $x = -2\frac{1}{3}$, or $x = 5$

(c) It should be clear from the graph that the LHS is equal to 5 for any value of $x \geq -\frac{1}{2}$. Solution: $x \geq -\frac{1}{2}$

8. 

$$\begin{aligned}
\angle \text{A} &= \cos^{-1}\frac{10^2+10^2-12^2}{2\times 10\times 10} \\
&\approx 1.287 \\
\angle \text{B} &= \cos^{-1}\frac{10^2+12^2-10^2}{2\times 10\times 12} \\
&\approx 0.927 \\
\angle \text{C} &= \angle \text{B} \\
&\approx 0.927 \\
a_{\triangle} &= \frac{1}{2}ab\sin \text{C} \\
&= \frac{1}{2}\times 12\times 10\sin 0.927 \\
&= 48 \\
a_{\text{sector A}} &= \frac{1}{2}\times 4^2\times 1.287 \\
&\approx 10.30 \\
a_{\text{sector B}} &= \frac{1}{2}\times 6^2\times 0.927 \\
&\approx 16.69 \\
a_{\text{sector C}} &= a_{\text{sector B}} \\
&\approx 16.69 \\
a_{\text{shaded}} &= a_{\triangle} - a_{\text{sector A}} - a_{\text{sector B}} - a_{\text{sector C}} \\
&= 48 - 10.30 - 2\times 16.69 \\
&\approx 4.3\text{cm}^2
\end{aligned}$$