Chapter 3

Exercise 3A

1. (a)
$$\begin{aligned}\angle ABN &= 180 - 50\\ &= 130°\\ \angle ABC &= 360 - 90 - 130\\ &= 140°\end{aligned}$$
$$\begin{aligned}AC &= \sqrt{5.8^2 + 6.4^2 - 2 \times 5.8 \times 6.4 \cos 140°}\\ &= 11.5\text{km}\end{aligned}$$
$$\begin{aligned}\frac{\sin \angle BAC}{6.4} &= \frac{\sin 140°}{11.5}\\ \angle BAC &= \sin^{-1} \frac{6.4 \sin 140°}{11.5}\\ &= 21°\\ 50 + 21 &= 71°\end{aligned}$$
C is 11.5km on a bearing of 071° from A.

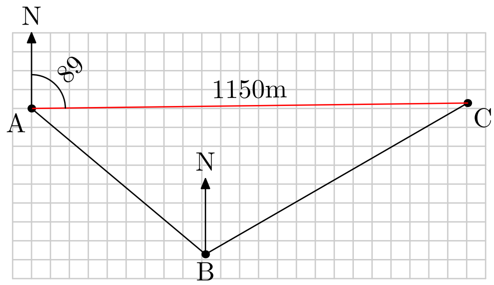
   (b) $71 + 180 = 251°$
   A has a bearing of 251° from C.

2. (a) Bearing of A from B is $300 - 180 = 120°$.
$$\begin{aligned}\angle ABC &= 120 - 70\\ &= 50°\end{aligned}$$
$$\begin{aligned}AC &= \sqrt{4.9^2 + 7.2^2 - 2 \times 4.9 \times 7.2 \cos 50°}\\ &= 5.5\text{km}\end{aligned}$$
We'll initially find ∠BCA rather than ∠BAC because the sine rule is ambiguous for ∠BAC but ∠BCA can not be obtuse (because it is opposite a smaller side).
$$\begin{aligned}\frac{\sin \angle BCA}{4.9} &= \frac{\sin 50°}{5.5}\\ \angle BCA &= \sin^{-1} \frac{4.9 \sin 50°}{5.5}\\ &= 43°\\ \angle BAC &= 180 - 50 - 43\\ &= 87°\\ 300 + 87 &= 387\\ 387 - 360 &= 027°\end{aligned}$$
C is 8.5km on a bearing of 027° from A.

   (b) $27 + 180 = 207°$
   A has a bearing of 207° from C.

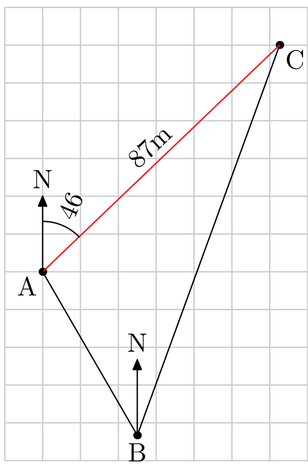
3. (a) Bearing of A from B is $40 + 180 = 220°$.
   Bearing of C from B is $360 - 100 = 260°$.
$$\begin{aligned}\angle ABC &= 260 - 220\\ &= 40°\end{aligned}$$
$$\begin{aligned}AC &= \sqrt{73^2 + 51^2 - 2 \times 73 \times 51 \cos 40°}\\ &= 47\text{km}\end{aligned}$$
We'll initially find ∠BCA rather than ∠BAC because the sine rule is ambiguous for ∠BAC but ∠BCA can not be obtuse (because it is opposite a smaller side).
$$\begin{aligned}\frac{\sin \angle BCA}{51} &= \frac{\sin 40°}{47}\\ \angle BCA &= \sin^{-1} \frac{51 \sin 40°}{47}\\ &= 44°\\ \angle BAC &= 180 - 40 - 44\\ &= 96°\\ 40 - 96 &= -56\\ -56 + 360 &= 304°\end{aligned}$$
C is 47km on a bearing of 304° from A.

   (b) $304 - 180 = 124°$
   A has a bearing of 124° from C.

4. (a)


Scale=1:20000

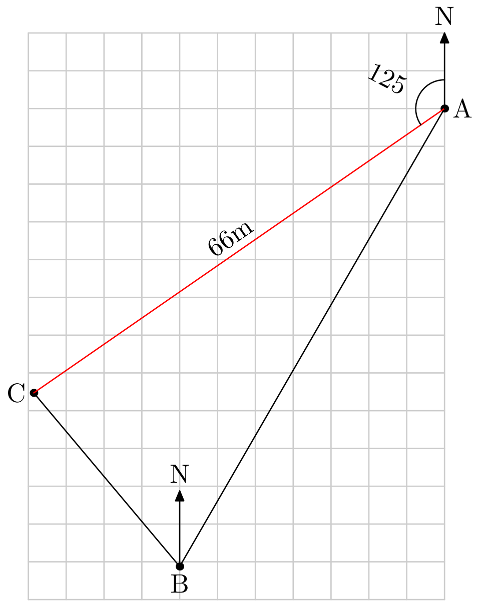
   (b) Bearing of A from C is $89 + 180 = 269°$

5. (a)


Scale=1:2000

   (b) Bearing of A from C is $46 + 180 = 226°$

6. (a)

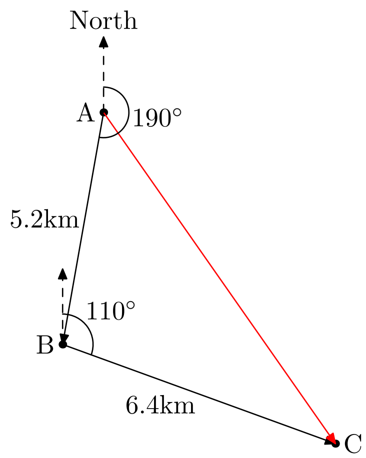


Scale=1:1000

Bearing of C from A is $360 - 125 = 235°$.

(b) Bearing of A from C is $215 - 180 = 055°$

7.



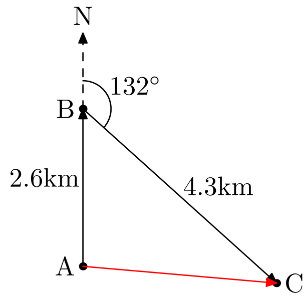
$$\begin{aligned}\angle ABC &= 110 - 10\\ &= 100°\end{aligned}$$

$$\begin{aligned}AC &= \sqrt{5.2^2 + 6.4^2 - 2 \times 5.2 \times 6.4 \cos 100°}\\ &= 8.9\text{km}\end{aligned}$$

$$\begin{aligned}\frac{\sin \angle BAC}{6.4} &= \frac{\sin 100°}{8.9}\\ \angle BAC &= \sin^{-1} \frac{6.4 \sin 100°}{8.9}\\ &= 45°\\ 190 - 45 &= 145°\end{aligned}$$

Final position is 8.9km on a bearing of 145° from initial position.

8.



$$\begin{aligned}\angle ABC &= 180 - 132\\ &= 48°\end{aligned}$$

$$\begin{aligned}AC &= \sqrt{2.6^2 + 4.3^2 - 2 \times 2.6 \times 4.3 \cos 48°}\\ &= 3.2\text{km}\end{aligned}$$

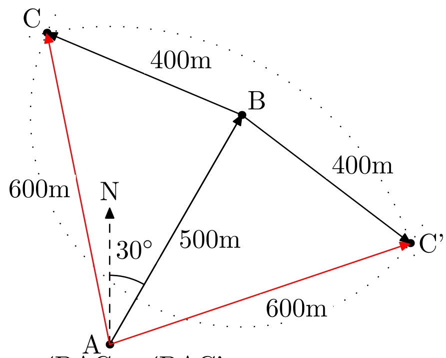
We'll initially find ∠BCA rather than ∠BAC because the sine rule is ambiguous for ∠BAC but ∠BCA can not be obtuse (because it is opposite a smaller side).

$$\begin{aligned}\frac{\sin \angle BCA}{2.6} &= \frac{\sin 48°}{3.2}\\ \angle BCA &= \sin^{-1} \frac{2.6 \sin 48°}{3.2}\\ &= 37°\\ \angle BCA &= 180 - 48 - 41\\ &= 95°\end{aligned}$$

Final position is 3.2km on a bearing of 095° from initial position.

9. $$\begin{aligned}d &= \sqrt{30^2 + 20^2 - 2 \times 30 \times 20 \cos 110}\\ &= 41\text{m}\end{aligned}$$

10.



Let θ = ∠BAC= ∠BAC'

$$\begin{aligned}400^2 &= 600^2 + 500^2 - 2 \times 600 \times 500 \cos \theta\\ \cos \theta &= \frac{600^2 + 500^2 - 400^2}{2 \times 600 \times 500}\\ \theta &= \cos^{-1} \frac{600^2 + 500^2 - 400^2}{2 \times 600 \times 500}\\ &= 41°\end{aligned}$$

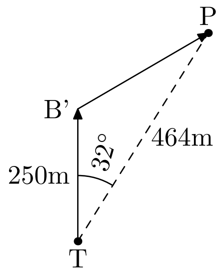
The bearing of the second checkpoint from the start is either: $(30-41)+360 = 349°$ or $30+41 = 071°$.

11. First, determine the bearing and distance from tee to pin. The angle at the bend is $180 - (50 - 20) = 150°$. Call the bend point B and tee and pin T and P respectively.

$$\begin{aligned} \text{TP} &= \sqrt{280^2 + 200^2 - 2 \times 280 \times 200 \cos 150°} \\ &= 464\text{m} \end{aligned}$$

$$\begin{aligned} \frac{\sin \angle \text{BTP}}{200} &= \frac{\sin 150°}{464} \\ \angle \text{BTP} &= \sin^{-1} \frac{200 \sin 150°}{464} \\ &= 12° \end{aligned}$$

So the pin is 464m from the tee on a bearing of $20 + 12 = 032°$. Now consider the result of the mis-hit:



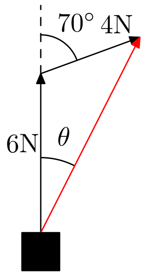
$$\begin{aligned} \text{B'P} &= \sqrt{250^2 + 464^2 - 2 \times 250 \times 464 \cos 32} \\ &= 286\text{m} \end{aligned}$$

We now need to find obtuse angle TB'P:

$$\begin{aligned} \frac{\sin \angle \text{TB'P}}{464} &= \frac{\sin 32°}{286} \\ \angle \text{TB'P} &= 180 - \sin^{-1} \frac{464 \sin 32°}{286} \\ &= 180 - 60° \end{aligned}$$

Hence the pin P is 286m from B' on a bearing of 060°.

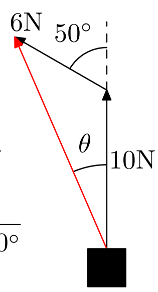
Exercise 3B



1. Let m be the magnitude of the resultant and θ the angle.

$$\begin{aligned} m &= \sqrt{6^2 + 4^2 - 2 \times 6 \times 4 \cos 110°} \\ &= 8.3 \end{aligned}$$

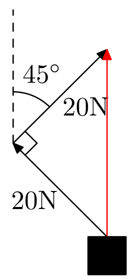
$$\begin{aligned} \frac{\sin \theta}{4} &= \frac{\sin 110°}{8.3} \\ \theta &= \sin^{-1} \frac{4 \sin 110°}{8.3} \\ &= 27° \end{aligned}$$



2. Let m be the magnitude of the resultant and θ the angle.

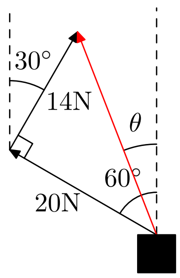
$$\begin{aligned} m &= \sqrt{10^2 + 8^2 - 2 \times 10 \times 8 \cos 130°} \\ &= 16.3 \end{aligned}$$

$$\begin{aligned} \frac{\sin \theta}{6} &= \frac{\sin 130°}{16.3} \\ \theta &= \sin^{-1} \frac{6 \sin 130°}{16.3} \\ &= 22° \end{aligned}$$



3. Let m be the magnitude of the resultant and θ the angle.

$$\begin{aligned} m &= \sqrt{20^2 + 20^2} \\ &= 28.3 \\ \theta &= 0 \end{aligned}$$

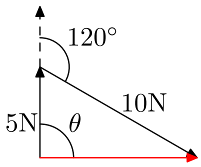


4. Let m be the magnitude of the resultant and θ the angle.

$$\begin{aligned} m &= \sqrt{14^2 + 20^2} \\ &= 24.4 \end{aligned}$$

$$\begin{aligned} \tan(60 - \theta) &= \frac{14}{20} \\ 60 - \theta &= 35 \\ \theta &= 25° \end{aligned}$$

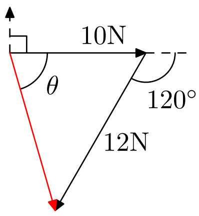
5. Let m be the magnitude of the resultant and θ the angle.



$$\begin{aligned} m &= \sqrt{5^2 + 10^2 - 2 \times 5 \times 10 \cos 60^\circ} \\ &= \sqrt{25 + 100 - 100 \times \frac{1}{2}} \\ &= \sqrt{75} \\ &= 5\sqrt{3} \\ \theta &= 090^\circ \end{aligned}$$

(We recognise it as a right angle triangle from our knowledge of exact trig ratios.)

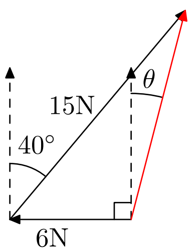
6. Let m be the magnitude of the resultant and θ as shown.



$$\begin{aligned} m &= \sqrt{12^2 + 10^2 - 2 \times 12 \times 10 \cos 60^\circ} \\ &= \sqrt{144 + 100 - 240 \times \frac{1}{2}} \\ &= \sqrt{124} \\ &= 2\sqrt{31} \\ \frac{\sin\theta}{12} &= \frac{\sin 60}{2\sqrt{31}} \\ \theta &= \sin^{-1} \frac{12 \sin 60}{2\sqrt{31}} \\ &= 69^\circ \end{aligned}$$

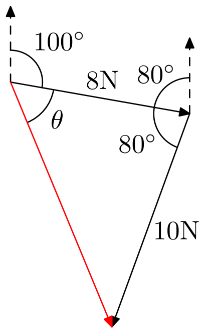
Bearing$=90 + 69 = 159^\circ$

7. Let m be the magnitude of the resultant and θ as shown.



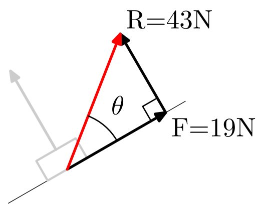
$$\begin{aligned} m &= \sqrt{6^2 + 15^2 - 2 \times 6 \times 15 \cos 50^\circ} \\ &= 12.1\text{N} \\ \frac{\sin(\phi)}{6} &= \frac{\sin 50}{12.1} \\ \phi &= \sin^{-1} \frac{6 \sin 50}{12.1} \\ &= 22^\circ \\ \theta &= 180 - 90 - 50 - 22^\circ \\ &= 018^\circ \end{aligned}$$

8. Let m be the magnitude of the resultant and θ as shown.



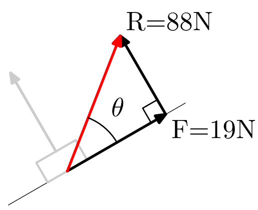
$$\begin{aligned} m &= \sqrt{8^2 + 10^2 - 2 \times 8 \times 10 \cos 80^\circ} \\ &= 11.7\text{N} \\ \frac{\sin\theta}{10} &= \frac{\sin 80}{11.7} \\ \theta &= \sin^{-1} \frac{10 \sin 80}{11.7} \\ &= 58^\circ \\ \text{bearing} &= 100 + 58^\circ \\ &= 158^\circ \end{aligned}$$

9.

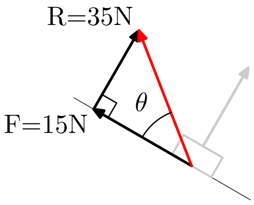


$$\begin{aligned} \text{magnitude} &= \sqrt{R^2 + F^2} \\ &= \sqrt{43^2 + 19^2} \\ &= 47\text{N} \\ \tan\theta &= \frac{R}{F} \\ \theta &= \tan^{-1} \frac{R}{F} \\ &= \tan^{-1} \frac{43}{19} \\ &= 66^\circ \end{aligned}$$

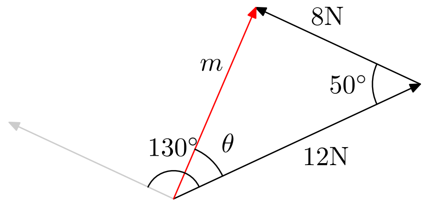
10.



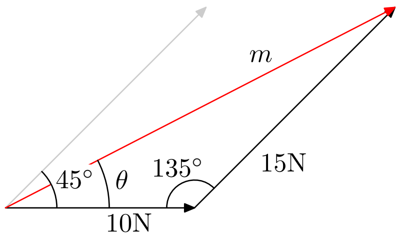
$$\begin{aligned} \text{magnitude} &= \sqrt{R^2 + F^2} \\ &= \sqrt{88^2 + 19^2} \\ &= 90\text{N} \\ \tan\theta &= \frac{R}{F} \\ \theta &= \tan^{-1} \frac{R}{F} \\ &= \tan^{-1} \frac{88}{19} \\ &= 78^\circ \end{aligned}$$

11. 

$$\begin{aligned} \text{magnitude} &= \sqrt{R^2 + F^2} \\ &= \sqrt{35^2 + 15^2} \\ &= 38\text{N} \\ \tan\theta &= \frac{R}{F} \\ \theta &= \tan^{-1}\frac{R}{F} \\ &= \tan^{-1}\frac{35}{15} \\ &= 67^\circ \end{aligned}$$

12. 

$$\begin{aligned} m &= \sqrt{8^2 + 12^2 - 2 \times 8 \times 12 \cos 50^\circ} \\ &= 9.2\text{N} \\ \frac{\sin\theta}{8} &= \frac{\sin 50^\circ}{9.2} \\ \theta &= \sin^{-1}\frac{8\sin 50^\circ}{9.2} \\ &= 42^\circ \end{aligned}$$

13. 

$$\begin{aligned} m &= \sqrt{10^2 + 15^2 - 2 \times 10 \times 15 \cos 135^\circ} \\ &= 23.2\text{N} \\ \frac{\sin\theta}{15} &= \frac{\sin 135^\circ}{23.2} \\ \theta &= \sin^{-1}\frac{15\sin 135^\circ}{23.2} \\ &= 27^\circ \end{aligned}$$

Exercise 3C

1. $$\begin{aligned} m &= \sqrt{2^2 + 4^2} \\ &= 4.5\text{m/s} \\ \tan\theta &= \frac{4}{2} \\ \theta &= 63^\circ \end{aligned}$$

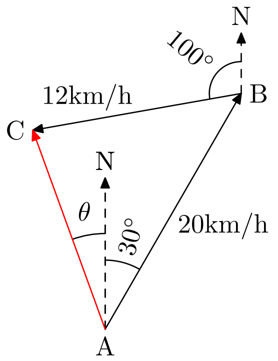
2. The angle formed where the vectors meet head to tail is $90 - 25 = 65^\circ$.

$$\begin{aligned} m &= \sqrt{2^2 + 4^2 - 2 \times 4 \times 2\cos 65} \\ &= 3.6\text{m/s} \\ \frac{\sin\theta}{4} &= \frac{\sin 65^\circ}{3.6} \\ \theta &= \sin^{-1}\frac{4\sin 65^\circ}{3.6} \\ &= 85^\circ \end{aligned}$$

3. The angle formed where the vectors meet head to tail is $180 - 50 = 130^\circ$.

$$\begin{aligned} m &= \sqrt{2^2 + 4^2 - 2 \times 4 \times 2\cos 130} \\ &= 5.5\text{m/s} \\ \frac{\sin\theta}{4} &= \frac{\sin 130^\circ}{5.5} \\ \theta &= \sin^{-1}\frac{4\sin 130^\circ}{5.5} \\ &= 34^\circ \end{aligned}$$

4.

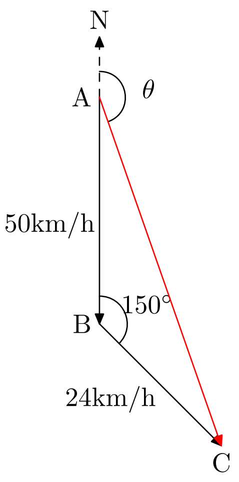


$$\begin{aligned}\angle\text{ABC} &= 180 - 30 - 100\\ &= 50^\circ\\ \text{AC} &= \sqrt{20^2 + 12^2 - 2\times 20\times 12\cos 50^\circ}\\ &= 15.3\text{km/h}\end{aligned}$$

$$\begin{aligned}\frac{\sin(\theta + 30^\circ)}{12} &= \frac{\sin 50^\circ}{15.3}\\ \theta + 30 &= \sin^{-1}\frac{12\sin 50^\circ}{15.3}\\ &= 37^\circ\\ \theta &= 7^\circ\end{aligned}$$

The boat travels on a bearing of 353° 15.3km in one hour.

5. Wind blowing *from* 330° is blowing *toward* 330 − 180 = 150°.

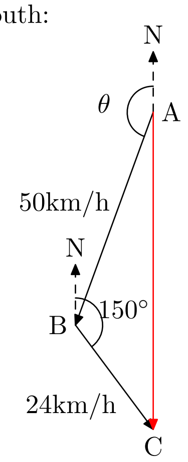


$$\begin{aligned}\text{AC} &= \sqrt{50^2 + 24^2 - 2\times 50\times 24\cos 150^\circ}\\ &= 71.8\text{km/h}\end{aligned}$$

$$\begin{aligned}\frac{\sin(180^\circ - \theta)}{24} &= \frac{\sin 150^\circ}{71.8}\\ 180 - \theta &= \sin^{-1}\frac{24\sin 150^\circ}{71.8}\\ &= 10^\circ\\ \theta &= 170^\circ\end{aligned}$$

The bird travels on a bearing of 170° at 71.8km/h.

To travel due south:



$$\begin{aligned}\angle\text{ACB} &= 180 - 150\\ &= 30^\circ\\ \frac{\sin(180^\circ - \theta)}{24} &= \frac{\sin 30^\circ}{50}\\ 180 - \theta &= \sin^{-1}\frac{24\sin 30^\circ}{50}\\ &= 14^\circ\\ \theta &= 166^\circ\end{aligned}$$

6. (a) $h = 3\times 60$

$= 180\text{m}$

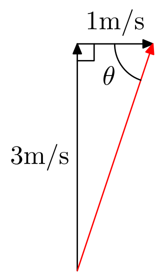
(b) $s = \sqrt{3^2 + 1^2}$

$= \sqrt{10}\,\text{m/s}$

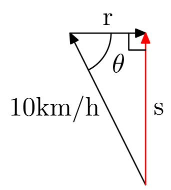
$\approx 3.2\,\text{m/s}$

(c) $\tan\theta = \dfrac{3}{1}$

$\theta = 72^\circ$



7. The angle can be determined using cosine. If r is the speed of the river current and θ is the angle with the bank, then $\cos\theta = \frac{r}{10}$.



The speed of the boat's movement across the river (s) can be determined using Pythagoras: $s = \sqrt{10^2 - r^2}$.

Then the time taken to cross the river is

$$t = \frac{0.08}{s}\times 3600 = \frac{288}{s}\ \text{seconds.}$$

(a) $\theta = \cos^{-1}\dfrac{3}{10}$

$= 73^\circ$

$s = \sqrt{10^2 - 3^2}$

$= 9.5\text{km/h}$

$t = \dfrac{288}{9.5}$

$= 30\,\text{s}$

(b) $\theta = \cos^{-1}\dfrac{4}{10}$

$= 66^\circ$

$s = \sqrt{10^2 - 4^2}$

$= 9.2\text{km/h}$

$t = \dfrac{288}{9.2}$

$= 31\,\text{s}$

(c) $\theta = \cos^{-1} \dfrac{6}{10}$

$$= 53°$$

$$s = \sqrt{10^2 - 6^2}$$

$$= 8\text{km/h}$$

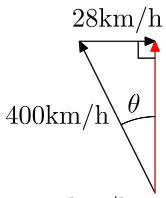
$$t = \frac{288}{8}$$

$$= 36\,\text{s}$$

8. $\sin\theta = \dfrac{28}{400}$

$$\theta = 4°$$

The plane should set a heading of N4°W or 356°T.

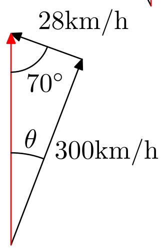


9. $\dfrac{\sin\theta}{28} = \dfrac{\sin 70°}{300}$

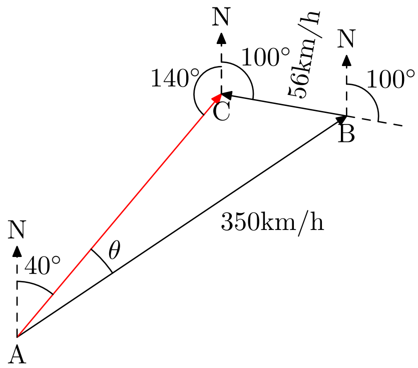
$$\theta = \sin^{-1}\frac{28\sin 70°}{300}$$

$$= 5°$$

The plane should set a heading of N5°E or 005°T.



10.



$$\angle ACB = 360 - 100 - 140$$

$$= 120°$$

$$\frac{\sin\theta}{56} = \frac{\sin 120°}{350}$$

$$\theta = \sin^{-1}\frac{56\sin 120°}{350}$$

$$= 8°$$

The plane should fly on a bearing of 048°.

$$\angle ABC = 180 - 120 - 8$$

$$= 52°$$

$$\frac{AC}{\sin 52°} = \frac{350}{\sin 120°}$$

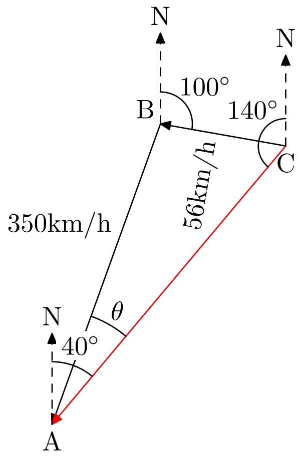
$$AC = \frac{350\sin 52°}{\sin 120°}$$

$$= 319\text{km/h}$$

Time required for the flight:

$$t = \frac{500}{319} \times 60 = 94 \text{ minutes}$$

For the return flight:



$$\angle ACB = 140 - 80$$

$$= 60°$$

$$\frac{\sin\theta}{56} = \frac{\sin 60°}{350}$$

$$\theta = \sin^{-1}\frac{56\sin 60°}{350}$$

$$= 8°$$

The plane should fly on a bearing of $180 + (40 - 8) = 212°$.

$$\angle ABC = 180 - 60 - 8$$

$$= 112°$$

$$\frac{AC}{\sin 112°} = \frac{350}{\sin 60°}$$

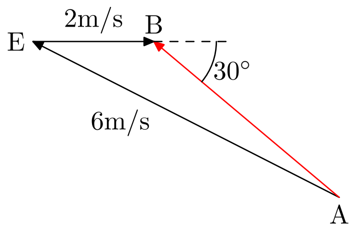
$$AC = \frac{350\sin 112°}{\sin 60°}$$

$$= 374\text{km/h}$$

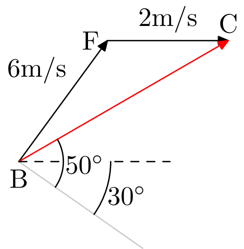
Time required for the return flight:

$$t = \frac{500}{374} \times 60 = 80 \text{ minutes}$$

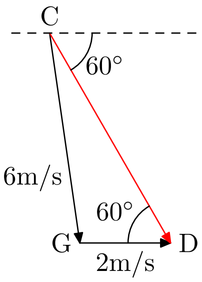
11.



$$\begin{aligned}\angle \text{B} &= 180 - 30\\ &= 150^\circ\\ \frac{\sin \angle A}{2} &= \frac{\sin 150^\circ}{6}\\ \angle A &= \sin^{-1}\frac{2\sin 150^\circ}{6}\\ &= 9.6^\circ\\ \angle E &= 180 - 150 - 9.6\\ &= 20.4^\circ\\ \frac{\text{AB}}{\sin 20.4^\circ} &= \frac{6}{\sin 150^\circ}\\ \text{AB} &= \frac{6\sin 20.4}{\sin 150^\circ}\\ &= 4.2\text{m/s}\\ t_{\text{AB}} &= \frac{80}{4.2}\\ &= 19.12\text{s}\end{aligned}$$



$$\begin{aligned}\angle \text{C} &= 50 - 30\\ &= 20^\circ\\ \frac{\sin \angle B}{2} &= \frac{\sin 20^\circ}{6}\\ \angle B &= \sin^{-1}\frac{2\sin 20^\circ}{6}\\ &= 6.5^\circ\\ \angle F &= 180 - 20 - 6.5\\ &= 153.5^\circ\\ \frac{\text{BC}}{\sin 153.5^\circ} &= \frac{6}{\sin 20^\circ}\\ \text{BC} &= \frac{6\sin 153.5}{\sin 20^\circ}\\ &= 7.8\text{m/s}\\ t_{\text{BC}} &= \frac{110}{7.8}\\ &= 14.03\text{s}\end{aligned}$$



$$\begin{aligned}\frac{\sin \angle C}{2} &= \frac{\sin 60^\circ}{6}\\ \angle C &= \sin^{-1}\frac{2\sin 60^\circ}{6}\\ &= 16.8^\circ\\ \angle G &= 180 - 60 - 16.8\\ &= 103.2^\circ\\ \frac{\text{BC}}{\sin 103.2^\circ} &= \frac{6}{\sin 60^\circ}\\ \text{BC} &= \frac{6\sin 103.2}{\sin 60^\circ}\\ &= 6.7\text{m/s}\end{aligned}$$

Perpendicular width of river:

$$\begin{aligned}w_{\text{AB}} &= 80\sin 30^\circ\\ &= 40\text{m}\\ w_{\text{BC}} &= 110\sin 20^\circ\\ &= 37.6\text{m}\\ w &= 40 + 37.6\\ &= 77.6\text{m}\\ \text{CD} &= \frac{77.6}{\sin 60^\circ}\\ &= 89.6\text{m}\\ t_{\text{CD}} &= \frac{89.6}{6.7}\\ &= 13.29\text{s}\end{aligned}$$

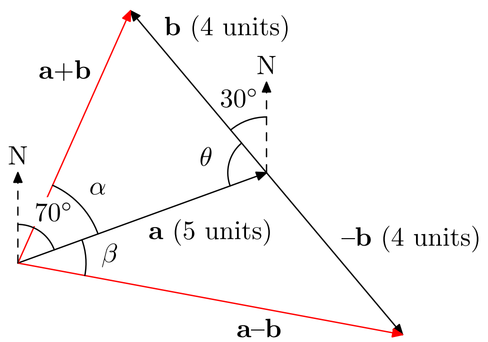
Total time:

$$\begin{aligned}t &= 19.12 + 14.03 + 13.29\\ &\approx 46\text{s}\end{aligned}$$

Exercise 3D

No working is needed for questions 1–7. Refer to the answers in Sadler.

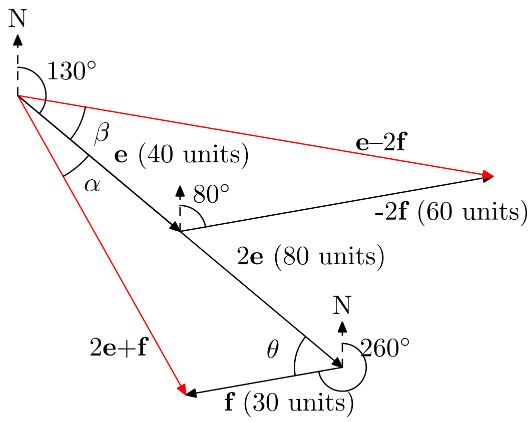
8.



(a)
$$\begin{aligned}\theta + 30 &= 180 - 70\\ \theta &= 80^\circ\\ |\mathbf{a}+\mathbf{b}| &= \sqrt{5^2+4^2-2\times5\times4\cos 80^\circ}\\ &= 5.8 \text{ units}\\ \frac{\sin\alpha}{4} &= \frac{\sin 80^\circ}{5.8}\\ \alpha &= \sin^{-1}\frac{4\sin 80^\circ}{5.8}\\ &= 42^\circ\\ 70-\alpha &= 28^\circ\end{aligned}$$

(b)
$$\begin{aligned}180-\theta &= 180-80\\ &= 100^\circ\\ |\mathbf{a}-\mathbf{b}| &= \sqrt{5^2+4^2-2\times5\times4\cos 100^\circ}\\ &= 6.9 \text{ units}\\ \frac{\sin\beta}{4} &= \frac{\sin 100^\circ}{6.9}\\ \beta &= \sin^{-1}\frac{4\sin 100^\circ}{6.9}\\ &= 35^\circ\\ 70+\beta &= 105^\circ\end{aligned}$$

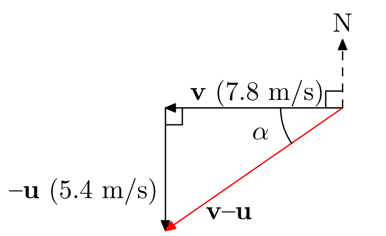
9.



(a)
$$\begin{aligned}\theta &= 360-260-(180-130)\\ &= 50^\circ\\ |2\mathbf{e}+\mathbf{f}| &= \sqrt{80^2+30^2-2\times80\times30\cos 50^\circ}\\ &= 65 \text{ units}\\ \frac{\sin\alpha}{30} &= \frac{\sin 50^\circ}{65}\\ \alpha &= \sin^{-1}\frac{30\sin 50^\circ}{65}\\ &= 21^\circ\\ 130+\alpha &= 151^\circ\end{aligned}$$

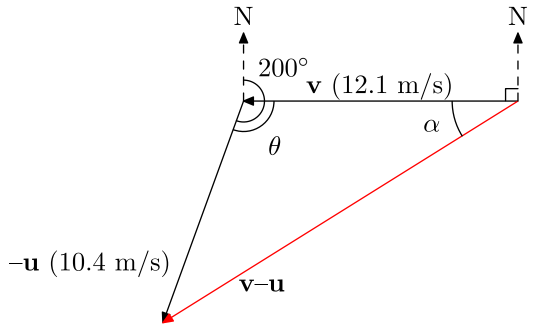
(b)
$$\begin{aligned}180-\theta &= 180-50\\ &= 130^\circ\\ |\mathbf{e}-2\mathbf{f}| &= \sqrt{40^2+60^2-2\times40\times60\cos 130^\circ}\\ &= 91 \text{ units}\\ \frac{\sin\beta}{60} &= \frac{\sin 130^\circ}{91}\\ \beta &= \sin^{-1}\frac{60\sin 130^\circ}{91}\\ &= 30^\circ\\ 130-\beta &= 100^\circ\end{aligned}$$

10.



$$\begin{aligned}|\mathbf{v}-\mathbf{u}| &= \sqrt{5.4^2+7.8^2}\\ &= 9.5 \text{ m/s}\\ \tan\alpha &= \frac{5.4}{7.8}\\ \alpha &= \tan^{-1}\frac{5.4}{7.8}\\ &= 35^\circ\\ 270-\alpha &= 235^\circ\\ \mathbf{a} &= \frac{\mathbf{v}-\mathbf{u}}{t}\\ &= \frac{9.5\angle 235^\circ}{5}\\ &= 1.9\,\text{m/s}^2 \text{ on a bearing of } 235^\circ\end{aligned}$$

11.



$$\theta = 200 - 90$$
$$= 110^\circ$$

$$|\mathbf{v} - \mathbf{u}| = \sqrt{10.4^2 + 12.1^2 - 2 \times 10.4 \times 12.1 \cos 110^\circ}$$
$$= 18.5 \text{ m/s}$$

$$\frac{\sin \alpha}{10.4} = \frac{\sin 110^\circ}{18.5}$$
$$\alpha = \sin^{-1} \frac{10.4 \sin 110^\circ}{18.5}$$
$$= 32^\circ$$
$$270 - \alpha = 238^\circ$$
$$\mathbf{a} = \frac{\mathbf{v} - \mathbf{u}}{t}$$
$$= \frac{18.5\angle 238^\circ}{4}$$
$$= 4.6\,\text{m/s}^2 \text{ on a bearing of } 238^\circ$$

12. (a) $\lambda = \mu = 0$

(b) $\lambda = \mu = 0$

(c) $\lambda - 3 = 0 \quad \mu + 4 = 0$

$\lambda = 3 \qquad \mu = -4$

(d) $(\lambda - 2)\mathbf{a} = (5 - \mu)\mathbf{b}$

$\lambda - 2 = 0 \qquad 5 - \mu = 0$

$\lambda = 2 \qquad \mu = 5$

(e) $\lambda\mathbf{a} - 2\mathbf{b} = \mu\mathbf{b} + 5\mathbf{a}$

$\lambda\mathbf{a} - 5\mathbf{a} = \mu\mathbf{b} + 2\mathbf{b}$

$(\lambda - 5)\mathbf{a} = (\mu + 2)\mathbf{b}$

$\lambda - 5 = 0 \qquad \mu + 2 = 0$

$\lambda = 5 \qquad \mu = -2$

(f) $(\lambda + \mu - 4)\mathbf{a} = (\mu - 3\lambda)\mathbf{b}$

$\lambda + \mu - 4 = 0 \qquad \mu - 3\lambda = 0$

$\mu = 4 - \lambda \qquad \mu = 3\lambda$

$4 - \lambda = 3\lambda$

$4 = 4\lambda$

$\lambda = 1$

$\mu = 3\lambda$

$\mu = 3$

(g) $2\mathbf{a} + 3\mathbf{b} + \mu\mathbf{b} = 2\mathbf{b} + \lambda\mathbf{a}$

$(2 - \lambda)\mathbf{a} = (2 - 3 - \mu)\mathbf{b}$

$2 - \lambda = 0 \qquad -1 - \mu = 0$

$\lambda = 2 \qquad \mu = -1$

(h) $\lambda\mathbf{a} + \mu\mathbf{b} + 2\lambda\mathbf{b} = 5\mathbf{a} + 4\mathbf{b} + \mu\mathbf{a}$

$(\lambda - 5 - \mu)\mathbf{a} = (4 - \mu - 2\lambda)\mathbf{b}$

$\lambda - 5 - \mu = 0$

$\mu = \lambda - 5$

$4 - \mu - 2\lambda = 0$

$4 - (\lambda - 5) - 2\lambda = 0$

$4 - \lambda + 5 - 2\lambda = 0$

$9 - 3\lambda = 0$

$\lambda = 3$

$\mu = \lambda - 5$

$\mu = -2$

(i) $\lambda\mathbf{a} - \mathbf{b} + \mu\mathbf{b} = 4\mathbf{a} + \mu\mathbf{a} - 4\lambda\mathbf{b}$

$(\lambda - 4 - \mu)\mathbf{a} = (-4\lambda + 1 - \mu)\mathbf{b}$

$\lambda - 4 - \mu = 0$

$\mu = \lambda - 4$

$-4\lambda + 1 - \mu = 0$

$-4\lambda + 1 - (\lambda - 4) = 0$

$-4\lambda + 1 - \lambda + 4 = 0$

$-5\lambda + 5 = 0$

$\lambda = 1$

$\mu = \lambda - 4$

$\mu = -3$

(j) $2\lambda\mathbf{a} + 3\mu\mathbf{a} - \mu\mathbf{b} + 2\mathbf{b} = \lambda\mathbf{b} + 2\mathbf{a}$

$(2\lambda + 3\mu - 2)\mathbf{a} = (\lambda + \mu - 2)\mathbf{b}$

$2\lambda + 3\mu - 2 = 0$

$\lambda + \mu - 2 = 0$

$2\lambda + 2\mu - 4 = 0$

$\mu + 2 = 0$

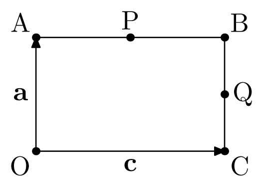
$\mu = -2$

$\lambda + \mu - 2 = 0$

$\lambda - 2 - 2 = 0$

$\lambda = 4$

13.



(a) $\overrightarrow{CB} = \mathbf{a}$

(b) $\overrightarrow{BC} = -\overrightarrow{CB} = -\mathbf{a}$

(c) $\overrightarrow{AB} = \mathbf{c}$

(d) $\overrightarrow{BA} = -\overrightarrow{AB} = -\mathbf{c}$

(e) $\overrightarrow{AP} = 0.5\overrightarrow{AB} = 0.5\mathbf{c}$

(f) $\overrightarrow{OQ} = \overrightarrow{OC} + \overrightarrow{CQ}$

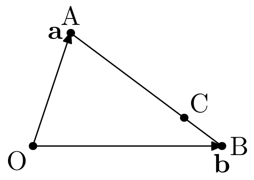
$= \mathbf{c} + 0.5\mathbf{a}$

(g) $\overrightarrow{OP} = \overrightarrow{OA} + \overrightarrow{AP}$

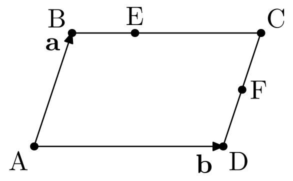
$= \mathbf{a} + 0.5\mathbf{c}$

(h) $\overrightarrow{PQ} = \overrightarrow{PB} + \overrightarrow{BQ}$

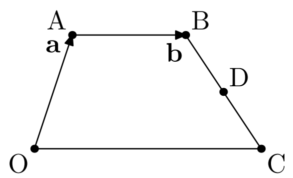
$= 0.5\mathbf{c} - 0.5\mathbf{a}$

14. 

(a) $\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB} = -\mathbf{a} + \mathbf{b}$
(b) $\overrightarrow{AC} = 0.75\overrightarrow{AB} = -0.75\mathbf{a} + 0.75\mathbf{b}$
(c) $\overrightarrow{CB} = 0.25\overrightarrow{AB} = -0.25\mathbf{a} + 0.25\mathbf{b}$
(d) $\overrightarrow{OC} = \overrightarrow{OA} + \overrightarrow{AC}$
$= \mathbf{a} - 0.75\mathbf{a} + 0.75\mathbf{b}$
$= 0.25\mathbf{a} + 0.75\mathbf{b}$

15. 

(a) $\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC} = \mathbf{a} + \mathbf{b}$
(b) $\overrightarrow{BE} = \frac{1}{3}\overrightarrow{BC} = \frac{1}{3}\mathbf{b}$
(c) $\overrightarrow{DF} = \frac{1}{2}\overrightarrow{DC} = \frac{1}{2}\mathbf{a}$
(d) $\overrightarrow{AE} = \overrightarrow{AB} + \overrightarrow{BE} = \mathbf{a} + \frac{1}{3}\mathbf{b}$
(e) $\overrightarrow{AF} = \overrightarrow{AD} + \overrightarrow{DF} = \mathbf{b} + \frac{1}{2}\mathbf{a}$
(f) $\overrightarrow{BF} = \overrightarrow{BA} + \overrightarrow{AF}$
$= -\mathbf{a} + \mathbf{b} + \frac{1}{2}\mathbf{a}$
$= \mathbf{b} - \frac{1}{2}\mathbf{a}$
(g) $\overrightarrow{DE} = \overrightarrow{DA} + \overrightarrow{AE}$
$= -\mathbf{b} + \mathbf{a} + \frac{1}{3}\mathbf{b}$
$= \mathbf{a} - \frac{2}{3}\mathbf{b}$
(h) $\overrightarrow{EF} = \overrightarrow{EA} + \overrightarrow{AF}$
$= -\overrightarrow{AE} + \overrightarrow{AF}$
$= -(\mathbf{a} + \frac{1}{3}\mathbf{b}) + \mathbf{b} + \frac{1}{2}\mathbf{a}$
$= -\frac{1}{2}\mathbf{a} + \frac{2}{3}\mathbf{b}$

16. 

(a) $\overrightarrow{OB} = \overrightarrow{OA} + \overrightarrow{AB} = \mathbf{a} + \mathbf{b}$
(b) $\overrightarrow{OC} = 2\overrightarrow{AB} = 2\mathbf{b}$
(c) $\overrightarrow{BC} = \overrightarrow{BA} + \overrightarrow{AO} + \overrightarrow{OC}$
$= -\overrightarrow{AB} - \overrightarrow{OA} + \overrightarrow{OC}$
$= -\mathbf{b} - \mathbf{a} + 2\mathbf{b}$
$= -\mathbf{a} + \mathbf{b}$
(d) $\overrightarrow{BD} = 0.5\overrightarrow{BC} = -0.5\mathbf{a} + 0.5\mathbf{b}$
(e) $\overrightarrow{OD} = \overrightarrow{OB} + \overrightarrow{BD}$
$= \mathbf{a} + \mathbf{b} - 0.5\mathbf{a} + 0.5\mathbf{b}$
$= 0.5\mathbf{a} + 1.5\mathbf{b}$

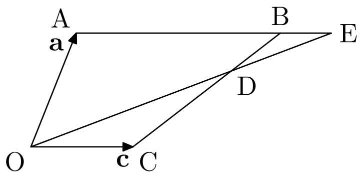
17. (a) $\overrightarrow{OC} = 0.5\overrightarrow{OA} = 0.5\mathbf{a}$

(b) $\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB} = -\mathbf{a} + \mathbf{b}$

(c) $\overrightarrow{AD} = \frac{2}{3}\overrightarrow{AB} = -\frac{2}{3}\mathbf{a} + \frac{2}{3}\mathbf{b}$

(d) $\overrightarrow{CD} = \overrightarrow{CA} + \overrightarrow{AD}$
$= \frac{1}{2}\mathbf{a} + (-\frac{2}{3}\mathbf{a} + \frac{2}{3}\mathbf{b})$
$= -\frac{1}{6}\mathbf{a} + \frac{2}{3}\mathbf{b}$

(e)
$$\begin{aligned}
\overrightarrow{OC} + \overrightarrow{CE} &= \overrightarrow{OE} \\
\overrightarrow{OC} + h\overrightarrow{CD} &= k\overrightarrow{OB} \\
\frac{1}{2}\mathbf{a} + h(-\frac{1}{6}\mathbf{a} + \frac{2}{3}\mathbf{b}) &= k\mathbf{b} \\
(\frac{1}{2} - \frac{h}{6})\mathbf{a} &= (k - \frac{2h}{3})\mathbf{b} \\
\frac{1}{2} - \frac{h}{6} &= 0 \\
3 - h &= 0 \\
h &= 3 \\
k - \frac{2h}{3} &= 0 \\
k &= \frac{2h}{3} \\
&= \frac{2 \times 3}{3} \\
&= 2
\end{aligned}$$

18. 

$$\begin{aligned}\overrightarrow{OD} &= \overrightarrow{OC} + \overrightarrow{CD}\\ &= \mathbf{c} + \frac{2}{3}\overrightarrow{CB}\\ &= \mathbf{c} + \frac{2}{3}(\overrightarrow{CO} + \overrightarrow{OA} + \overrightarrow{AB})\\ &= \mathbf{c} + \frac{2}{3}(-\mathbf{c} + \mathbf{a} + 2\mathbf{c})\\ &= \mathbf{c} + \frac{2}{3}(\mathbf{a} + \mathbf{c})\\ &= \frac{2}{3}\mathbf{a} + \frac{5}{3}\mathbf{c}\end{aligned}$$

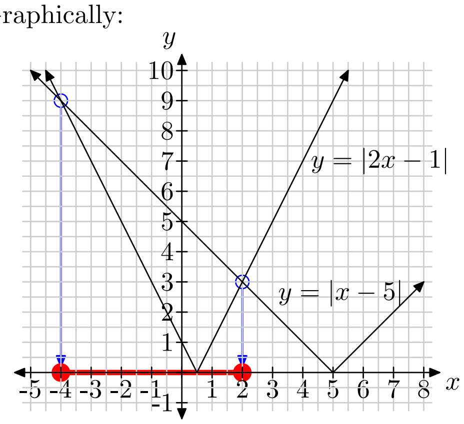
$$\begin{aligned}\overrightarrow{OE} &= \overrightarrow{OA} + \overrightarrow{AE}\\ h\overrightarrow{OD} &= \overrightarrow{OA} + k\overrightarrow{AB}\\ h(\frac{2}{3}\mathbf{a} + \frac{5}{3}\mathbf{c}) &= \mathbf{a} + 2k\mathbf{c}\\ \frac{2h}{3}\mathbf{a} + \frac{5h}{3}\mathbf{c} &= \mathbf{a} + 2k\mathbf{c}\\ (\frac{2h}{3} - 1)\mathbf{a} &= (2k - \frac{5h}{3})\mathbf{c}\end{aligned}$$

$$\begin{aligned}\frac{2h}{3} - 1 &= 0\\ \frac{2h}{3} &= 1\\ 2h &= 3\\ h &= \frac{3}{2}\end{aligned}$$

$$\begin{aligned}2k - \frac{5h}{3} &= 0\\ 2k &= \frac{5h}{3}\\ k &= \frac{5h}{6}\\ &= \frac{5}{6} \times \frac{3}{2}\\ &= \frac{5}{4}\end{aligned}$$

Miscellaneous Exercise 3

1. (a) Graphically:



$$-4 \le x \le 2$$

Algebraically:
First solve $|2x - 1| = |x - 5|$

$2x - 1 = x - 5$ or $-(2x - 1) = x - 5$

$x = -4$ $-2x + 1 = x - 5$

$-3x = -4$

$x = 2$

Now test one of the three intervals delimited by these two solutions. Try a value, say $x = 0$:
Is it true that $|5(0) - 1| \le |(0) - 5|$?
Yes $(1 \le 5)$.

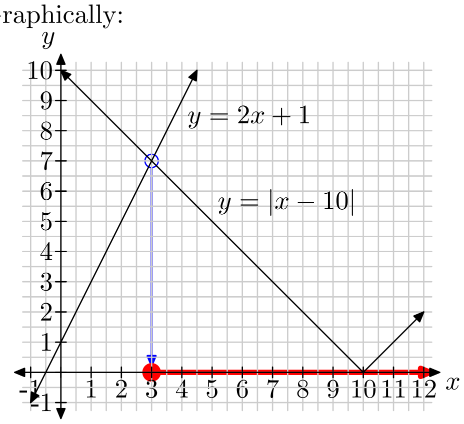
Solution set is

$$\{x \in \mathbb{R} : -4 \le x \le 2\}$$

(b) This is the complementary case to the previous question, so it has the complementary solution:

$$\{x \in \mathbb{R} : x < -4\} \cup \{x \in \mathbb{R} : x > 2\}$$

(c) Graphically:



$$x \geq 3$$

Algebraically:
First solve $|x - 10| = 2x + 1$

$$\begin{aligned} x - 10 &= 2x + 1 \quad \text{or} \quad & -(x - 10) &= 2x + 1 \\ x &= -11 & -x + 10 &= 2x + 1 \\ & & -3x &= -9 \\ & & x &= 3 \end{aligned}$$

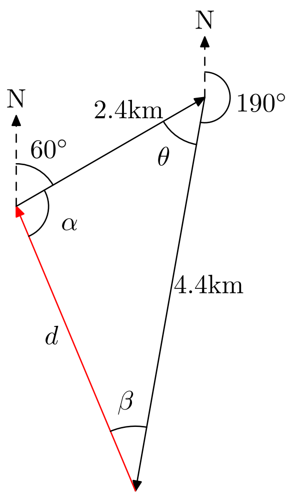
However, $x = -11$ is not actually a solution, as you can see by substituting into the equation, so we are left with two intervals (either side of $x = 3$).

Now test one of these intervals delimited by these two solutions. Try a value, say $x = 0$:
Is it true that $|(0) - 10| \leq 2(0) + 1$?
No ($10 \not\leq 1$).

Solution set is

$$\{x \in \mathbb{R} : x \geq 3\}$$

2.



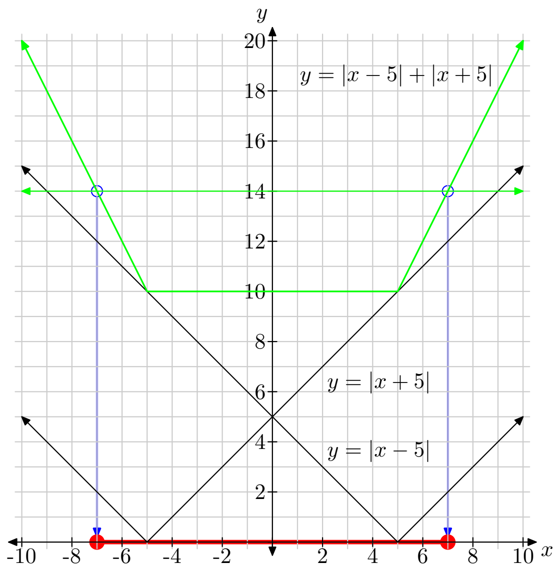
$$\begin{aligned} \theta &= 369 - 190 - (180 - 60) \\ &= 50° \end{aligned}$$

$$\begin{aligned} d &= \sqrt{2.4^2 + 4.4^2 - 2 \times 2.4 \times 4.4 \cos 50°} \\ &= 3.4\text{km} \end{aligned}$$

It's tempting to find angle α using the sine rule, but because it's opposite the longest side of the triangle, it could be either acute or obtuse: it's the ambiguous case. Finding β instead is unambiguous. β can not be obtuse because it is opposite a shorter side.

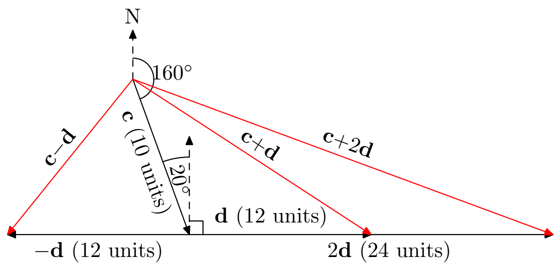
$$\begin{aligned} \frac{\sin \beta}{2.4} &= \frac{\sin 50°}{3.4} \\ \beta &= \sin^{-1} \frac{2.4 \sin 50°}{3.4} \\ &= 33° \\ \text{bearing} &= 190 + (180 - 33) \\ &= 327° \end{aligned}$$

3.



$|x - 5| + |x + 5| \leq 14$ for $\{x \in \mathbb{R} : -7 \leq x \leq 7\}$

4.



In each case below, let θ be the angle formed between **c** and the resultant.

(a) $$\begin{aligned} |\mathbf{c} + \mathbf{d}| &= \sqrt{10^2 + 12^2 - 2 \times 10 \times 12 \cos 110°} \\ &= 18.1 \text{ units} \end{aligned}$$

$$\begin{aligned} \frac{\sin \theta}{12} &= \frac{\sin 110°}{18.1} \\ \theta &= \sin^{-1} \frac{12 \sin 110°}{18.1} \\ &= 39° \\ \text{direction} &= 160 - 39 \\ &= 121° \end{aligned}$$

(b) $|\mathbf{c} - \mathbf{d}| = \sqrt{10^2 + 12^2 - 2 \times 10 \times 12 \cos 70°}$
$= 12.7$ units

$$\frac{\sin\theta}{12} = \frac{\sin 70°}{12.7}$$
$$\theta = \sin^{-1}\frac{12\sin 70°}{12.7}$$
$$= 62°$$
$$\text{direction} = 160 + 62$$
$$= 222°$$

(c) $|\mathbf{c} + 2\mathbf{d}| = \sqrt{10^2 + 24^2 - 2 \times 10 \times 24 \cos 110°}$
$= 29.0$ units

$$\frac{\sin\theta}{24} = \frac{\sin 110°}{29.0}$$
$$\theta = \sin^{-1}\frac{24\sin 110°}{29.0}$$
$$= 51°$$
$$\text{direction} = 160 - 51$$
$$= 109°$$

5. First, rearrange the equation to

$$|x - a| + |x + 3| = 5$$

and read this as "distance from a plus distance from -3 is equal to 5".

- If the distance between a and -3 is greater than 5 then the equation has no solution.
- If the distance between a and -3 is equal to 5 then every point between a and -3 is a solution.
- If the distance between a and -3 is less than 5 then there will be two solutions, one lying above the interval between -3 and a and one lying below it.

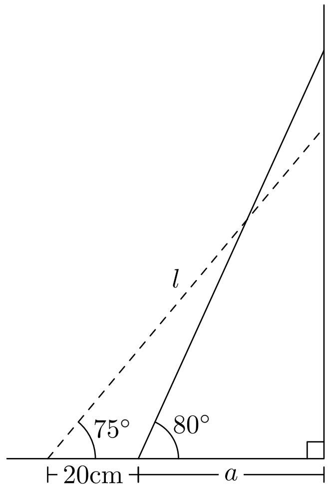
(a) For exactly two solutions,

$$|a + 3| < 5$$
$$-5 < a + 3 < 5$$
$$-8 < a < 2$$

(b) For more than two solutions,

$$|a + 3| = 5$$
$$a + 3 = 5 \quad \text{or} \quad a + 3 = -5$$
$$a = 2 \qquad\qquad a = -8$$

6. Let l be the length of the ladder.



$$\cos 80° = \frac{a}{l}$$
$$a = l\cos 80°$$
$$\cos 75° = \frac{a + 20}{l}$$
$$a + 20 = l\cos 75°$$
$$a = l\cos(75°) - 20$$
$$l\cos 80° = l\cos(75°) - 20$$
$$l\cos(75°) - l\cos 80° = 20$$
$$l(\cos(75°) - \cos 80°) = 20$$
$$l = \frac{20}{\cos(75°) - \cos 80°}$$
$$= 235\text{cm}$$
$$a = l\cos 80°$$
$$= 41\text{cm}$$

7. (a) $h = k = 0$

(b) $h\mathbf{a} + \mathbf{b} = k\mathbf{b}$
$h\mathbf{a} = k\mathbf{b} - \mathbf{b}$
$= (k - 1)\mathbf{b}$
$h = 0 \qquad k - 1 = 0$
$k = 1$

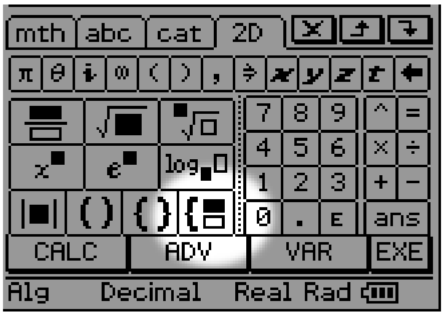
(c) $(h - 3)\mathbf{a} = (k + 1)\mathbf{b}$
$h - 3 = 0 \qquad k + 1 = 0$
$h = 3 \qquad k = -1$

(d) $h\mathbf{a} + 2\mathbf{a} = k\mathbf{b} - 3\mathbf{a}$
$h\mathbf{a} + 5\mathbf{a} = k\mathbf{b}$
$(h + 5)\mathbf{a} = k\mathbf{b}$
$h + 5 = 0 \qquad k = 0$
$h = -5$

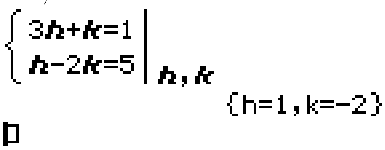
(e) $3h\mathbf{a} + k\mathbf{a} + h\mathbf{b} - 2k\mathbf{b} = \mathbf{a} + 5\mathbf{b}$
$3h\mathbf{a} + k\mathbf{a} - \mathbf{a} = 5\mathbf{b} - h\mathbf{b} + 2k\mathbf{b}$
$(3h + k - 1)\mathbf{a} = (5 - h + 2k)\mathbf{b}$

$3h + k - 1 = 0 \qquad 5 - h + 2k = 0$
$3h + k = 1 \qquad h - 2k = 5$
$h = 1$
$k = -2$

(Note: the final step in the solution above is done by solving the simultaneous equations $3h + k = 1$ and $h - 2k = 5$. You should be familiar with doing this by elimination or substitution. (Either would be suitable here.) You should also know how to do it on the ClassPad:



In the Main application, select the simultaneous equations icon in the 2D tab. Enter the two equations to the left of the vertical bar, and the two variables to the right:



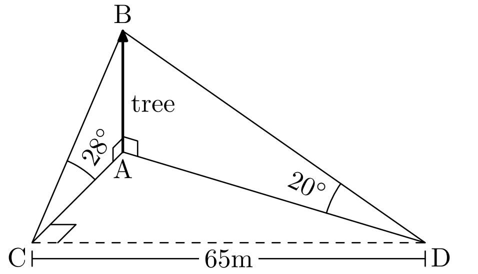
(f) $h(\mathbf{a} + \mathbf{b}) + k(\mathbf{a} - \mathbf{b}) = 3\mathbf{a} + 5\mathbf{b}$

$$(h + k)\mathbf{a} + (h - k)\mathbf{b} = 3\mathbf{a} + 5\mathbf{b}$$
$$(h + k - 3)\mathbf{a} = -(h - k - 5)\mathbf{b}$$
$$h + k - 3 = 0 \qquad h - k - 5 = 0$$
$$h + k = 3 \qquad h - k = 5$$

solving by elimination:

$$2h = 8$$
$$h = 4$$
$$4 + k = 3$$
$$k = -1$$

8.



Let the height of the tree be h. Let A be the point at the base of the tree and B the point at the apex.

$$\tan 28° = \frac{h}{\text{AC}}$$
$$\text{AC} = \frac{h}{\tan 28°}$$
$$\tan 20° = \frac{h}{\text{AD}}$$
$$\text{AD} = \frac{h}{\tan 20°}$$

$\triangle$ACD is right-angled at C, so

$$\text{AD}^2 = \text{AC}^2 + \text{CD}^2$$
$$\frac{h^2}{\tan^2 20°} = \frac{h^2}{\tan^2 28°} + 65^2$$
$$h^2\left(\frac{1}{\tan^2 20°} - \frac{1}{\tan^2 28°}\right) = 65^2$$

Solving this and discarding the negative root:

$$h = 32.5\text{m}$$
$$\text{AC} = \frac{h}{\tan 28°}$$
$$= 61.0\text{m}$$