Chapter 2

Exercise 2A

1. (a) 035° (read directly from the diagram)

   (b) $35 + 45 = 080°$

   (c) $35 + 45 + 30 = 110°$

   (d) $180 - 35 = 145°$

   (e) $180 + 20 = 200°$

   (f) $360 - 60 = 300°$

   (g) Back bearings:
   $35 + 180 = 215°$

   (h) $80 + 180 = 260°$

   (i) $110 + 180 = 290°$

   (j) $145 + 180 = 325°$

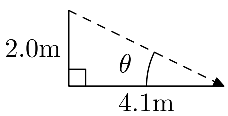
   (k) $200 - 180 = 020°$

   (l) $300 - 180 = 120°$

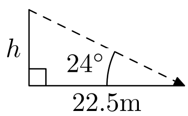
2. No working required. Refer to the answers in Sadler.

3. $$\begin{aligned}\tan 28° &= \frac{h}{22.4}\\ h &= 22.4\tan 28°\\ &= 11.9\text{m}\end{aligned}$$

4. $$\begin{aligned}\tan\theta &= \frac{2}{4.1}\\ \theta &= \tan^{-1}\frac{2}{4.1}\\ &= 26°\end{aligned}$$

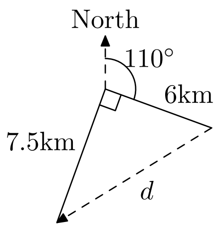


5. $$\begin{aligned}\tan 24° &= \frac{h}{22.5}\\ h &= 22.5\tan 24°\\ &= 10.0\text{m}\end{aligned}$$

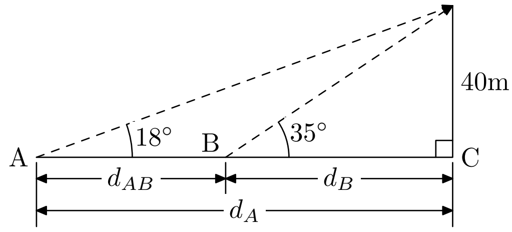


6. After one and a half hours, the first ship has travelled 6km and the second 7.5km.

$$\begin{aligned}d^2 &= 6^2 + 7.5^2\\ d &= 9.6\text{km}\\ &\approx 10\text{km}\end{aligned}$$

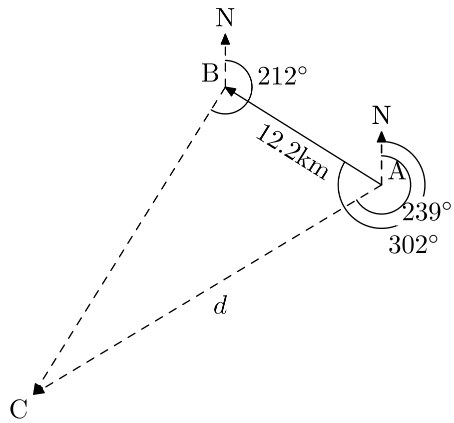


7.



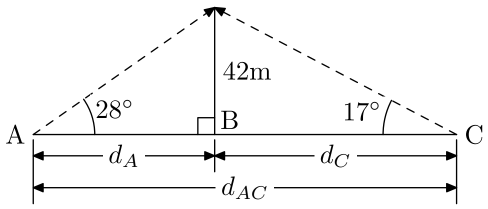
$$\begin{aligned}\tan 18° &= \frac{40}{d_A}\\ d_A &= \frac{40}{\tan 18°}\\ \tan 35° &= \frac{40}{d_B}\\ d_B &= \frac{40}{\tan 35°}\\ d_{AB} &= \frac{40}{\tan 18°} - \frac{40}{\tan 35°}\\ &= 66\text{m}\end{aligned}$$

8.



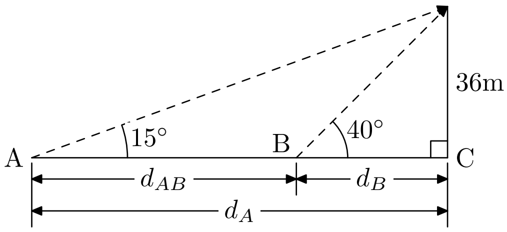
$$\begin{aligned}\angle\text{CAB} &= 302 - 239\\ &= 63°\\ \angle\text{CBA} &= 212 - (302 - 180)\\ &= 90°\\ \cos 63° &= \frac{12.2}{d}\\ d &= \frac{12.2}{\cos 63°}\\ &= 26.9\text{km}\end{aligned}$$

9.



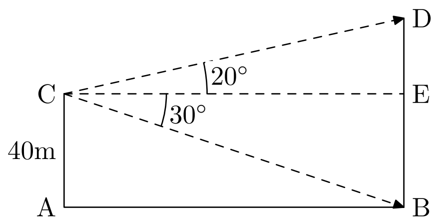
$$\begin{aligned}\tan 28^\circ &= \frac{42}{d_A}\\ d_A &= \frac{42}{\tan 28^\circ}\\ \tan 17^\circ &= \frac{42}{d_C}\\ d_C &= \frac{42}{\tan 17^\circ}\\ d_{AC} &= \frac{42}{\tan 28^\circ} + \frac{42}{\tan 17^\circ}\\ &= 216\text{m}\end{aligned}$$

10.



$$\begin{aligned}\tan 15^\circ &= \frac{36}{d_A}\\ d_A &= \frac{36}{\tan 15^\circ}\\ \tan 40^\circ &= \frac{36}{d_B}\\ d_B &= \frac{36}{\tan 40^\circ}\\ d_{AB} &= \frac{36}{\tan 15^\circ} - \frac{36}{\tan 40^\circ}\\ &= 91\text{m}\end{aligned}$$

11.



Distance between towers:

$$\begin{aligned}\tan 30^\circ &= \frac{40}{\text{AB}}\\ \text{AB} &= \frac{40}{\tan 30^\circ}\\ &= 40\sqrt{3}\end{aligned}$$

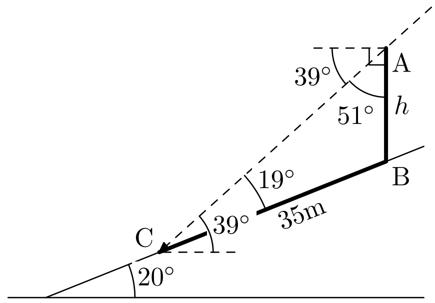
Additional height of second tower:

$$\begin{aligned}\tan 20^\circ &= \frac{\text{DE}}{40\sqrt{3}}\\ \text{DE} &= 40\sqrt{3}\tan 20\\ &= 25.22\text{m}\end{aligned}$$

Total height of second tower:

$$\begin{aligned}\text{DB} &= 25.22 + 40\\ &\approx 65.2\text{m}\end{aligned}$$

12.



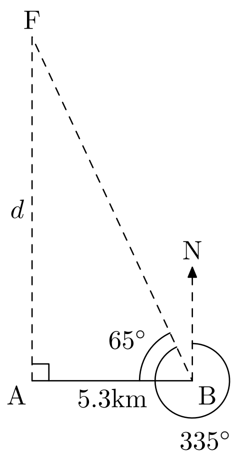
First determine the angles in the triangle made by the tree, the hillslope and the sun's ray.

$$\begin{aligned}\angle\text{ACB} &= 39 - 20\\ &= 19^\circ\\ \angle\text{CAB} &= 90 - 39\\ &= 51^\circ\end{aligned}$$

Now use the sine rule:

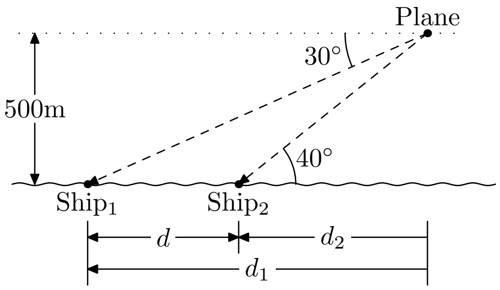
$$\begin{aligned}\frac{h}{\sin 19^\circ} &= \frac{35}{\sin 51^\circ}\\ h &= \frac{35\sin 19^\circ}{\sin 51^\circ}\\ &= 14.7\text{m}\end{aligned}$$

13.



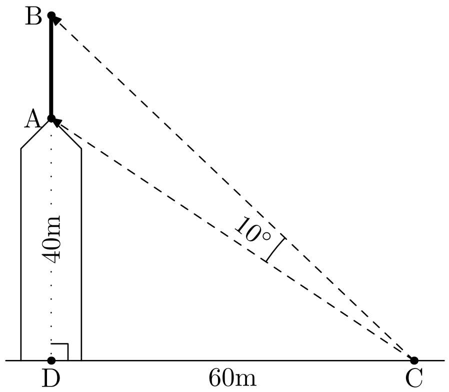
$$\begin{aligned}\angle\text{ABF} &= 335 - 270\\ &= 65^\circ\\ \tan 65^\circ &= \frac{d}{5.3}\\ d &= 5.3\tan 65^\circ\\ &= 11.4\text{km}\end{aligned}$$

14.



$$\begin{aligned}\tan 30^\circ &= \frac{500}{d_1}\\ d_1 &= \frac{500}{\tan 30^\circ}\\ &= 866\text{m}\\ \tan 40^\circ &= \frac{500}{d_2}\\ d_2 &= \frac{500}{\tan 40^\circ}\\ &= 596\text{m}\\ d &= d_1 - d_2\\ &= 270\text{m}\end{aligned}$$

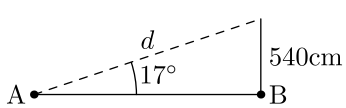
15.



$$\begin{aligned}\text{AC} &= \sqrt{60^2+40^2}\\ &= 20\sqrt{13}\\ \tan\angle\text{DAC} &= \frac{60}{40}\\ \angle\text{DAC} &= 56.3^\circ\\ \angle\text{BAC} &= 180-56.3\\ &= 123.7^\circ\\ \angle\text{ABC} &= 180-123.7-10\\ &= 46.3^\circ\\ \frac{\text{AB}}{\sin 10^\circ} &= \frac{20\sqrt{13}}{\sin 46.3^\circ}\\ \text{AB} &= \frac{20\sqrt{13}\sin 10^\circ}{\sin 46.3^\circ}\\ &= 17.3\text{m}\end{aligned}$$

16.

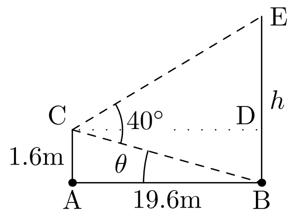
$$\begin{aligned}\sin 17^\circ &= \frac{540}{d}\\ d &= \frac{540}{\sin 17^\circ}\\ &= 1847\text{cm}\end{aligned}$$



Rounded up to the next metre this is 19m.

17.

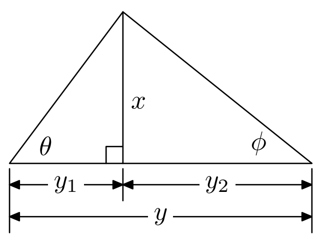
$$\begin{aligned}\tan\theta &= \frac{1.6}{19.6}\\ \theta &= 4.7^\circ\\ \angle\text{DCE} &= 40-4.7\\ &= 35.3^\circ\\ \tan 35.3^\circ &= \frac{\text{DE}}{19.6}\\ \text{DE} &= 19.6\tan 35.3^\circ\\ &= 13.9\text{m}\\ h &= 13.9+1.6\\ &= 15.5\text{m}\end{aligned}$$



18. Let the height of the flagpole be h and the distance from the base be d. Let θ be the angle of elevation of the point $\frac{3}{4}$ of the way up the flagpole.

$$\begin{aligned}\tan 40^\circ &= \frac{h}{d}\\ \tan\theta &= \frac{0.75h}{d}\\ &= 0.75\frac{h}{d}\\ &= 0.75\tan 40^\circ\\ \theta &= \tan^{-1}(0.75\tan 40^\circ)\\ &= 32^\circ\end{aligned}$$

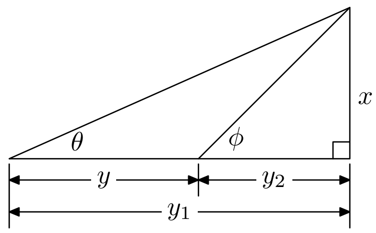
19.



$$\begin{aligned}\tan\theta &= \frac{x}{y_1}\\ y_1 &= \frac{x}{\tan\theta}\\ \tan\phi &= \frac{x}{y_2}\\ y_2 &= \frac{x}{\tan\phi}\\ y &= y_1+y_2\\ &= \frac{x}{\tan\theta}+\frac{x}{\tan\phi}\\ &= x\left(\frac{1}{\tan\theta}+\frac{1}{\tan\phi}\right)\\ &= x\left(\frac{\tan\phi}{\tan\theta\tan\phi}+\frac{\tan\theta}{\tan\theta\tan\phi}\right)\\ &= x\left(\frac{\tan\theta+\tan\phi}{\tan\theta\tan\phi}\right)\end{aligned}$$

□

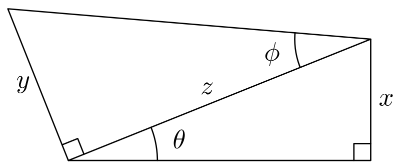
20.



$$\begin{aligned}
\tan\theta &= \frac{x}{y_1}\\
y_1 &= \frac{x}{\tan\theta}\\
\tan\phi &= \frac{x}{y_2}\\
y_2 &= \frac{x}{\tan\phi}\\
y &= y_1 - y_2\\
&= \frac{x}{\tan\theta} - \frac{x}{\tan\phi}\\
&= x\left(\frac{1}{\tan\theta} - \frac{1}{\tan\phi}\right)\\
&= x\left(\frac{\tan\phi}{\tan\theta\tan\phi} - \frac{\tan\theta}{\tan\theta\tan\phi}\right)\\
&= x\left(\frac{\tan\phi - \tan\theta}{\tan\theta\tan\phi}\right)
\end{aligned}$$

□

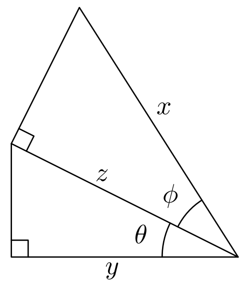
21.



$$\begin{aligned}
\sin\theta &= \frac{x}{z}\\
z &= \frac{x}{\sin\theta}\\
\tan\phi &= \frac{y}{z}\\
y &= z\tan\phi\\
&= \left(\frac{x}{\sin\theta}\right)\tan\phi\\
&= \frac{x\tan\phi}{\sin\theta}
\end{aligned}$$

□

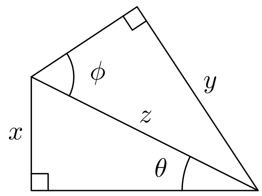
22.



$$\begin{aligned}
\cos\phi &= \frac{z}{x}\\
z &= \frac{x}{\cos\phi}\\
\cos\theta &= \frac{y}{z}\\
y &= z\cos\theta\\
&= (x\cos\phi)\cos\theta\\
&= x\cos\phi\cos\theta
\end{aligned}$$

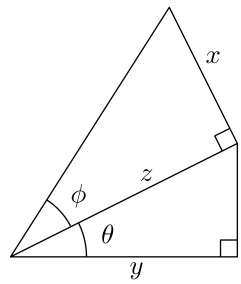
□

23. (a)



$$\begin{aligned}
\sin\theta &= \frac{x}{z}\\
z &= \frac{x}{\sin\theta}\\
\sin\phi &= \frac{y}{z}\\
y &= z\sin\phi\\
&= \left(\frac{x}{\sin\theta}\right)\sin\phi\\
&= \frac{x\sin\phi}{\sin\theta}
\end{aligned}$$

(b)



$$\begin{aligned}
\tan\phi &= \frac{x}{z}\\
z &= \frac{x}{\tan\phi}\\
\cos\theta &= \frac{y}{z}\\
y &= z\cos\theta\\
&= \left(\frac{x}{\tan\phi}\right)\cos\theta\\
&= \frac{x\cos\theta}{\tan\phi}
\end{aligned}$$

Exercise 2B

1. (a)
$$\begin{aligned} AM &= \frac{1}{2}AC \\ &= \frac{1}{2}\sqrt{AB^2 + BC^2} \\ &= \frac{1}{2}\sqrt{5.2^2 + 5.2^2} \\ &= 3.68\text{cm} \end{aligned}$$

(b)
$$\begin{aligned} \tan\angle EAM &= \frac{EM}{AM} \\ \angle EAM &= \tan^{-1}\frac{EM}{AM} \\ &= \tan^{-1}\frac{6.3}{3.68} \\ &= 59.7° \end{aligned}$$

(c)
$$\begin{aligned} \angle DEM &= \angle AEM \\ &= 180 - 90 - \angle EAM \\ &= 90 - 59.7 \\ &= 30.3° \end{aligned}$$

(d) Let F be the midpoint of AB. The angle between the face EAB and the base ABCD is $\angle EFM$.
$$\begin{aligned} FM &= \frac{1}{2}AB \\ &= 2.6\text{cm} \end{aligned}$$
$$\begin{aligned} \tan\angle EFM &= \frac{EM}{FM} \\ \angle EFM &= \tan^{-1}\frac{EM}{FM} \\ &= \tan^{-1}\frac{6.3}{2.6} \\ &= 67.6° \end{aligned}$$

2. (a)
$$\begin{aligned} \tan\angle GDC &= \frac{GC}{DC} \\ \angle GDC &= \tan^{-1}\frac{GC}{DC} \\ &= \tan^{-1}\frac{35}{62} \\ &= 29.4° \end{aligned}$$

(b)
$$\begin{aligned} \tan\angle GBC &= \frac{GC}{BC} \\ \angle GBC &= \tan^{-1}\frac{GC}{BC} \\ &= \tan^{-1}\frac{35}{38} \\ &= 42.6° \end{aligned}$$

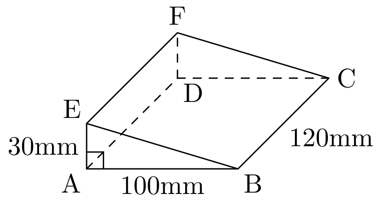
(c)
$$\begin{aligned} \tan\angle GAC &= \frac{GC}{AC} \\ \angle GAC &= \tan^{-1}\frac{GC}{AC} \\ &= \tan^{-1}\frac{35}{\sqrt{62^2 + 38^2}} \\ &= 25.7° \end{aligned}$$

(d)
$$\begin{aligned} AG &= \sqrt{AC^2 + GC^2} \\ &= \sqrt{62^2 + 38^2 + 35^2} \\ &= 80.7\text{mm} \end{aligned}$$

(e) The angle between the plane FADG and the base ABCD is equal to $\angle GDC = 29.4°$.

(f) The angle between skew lines DB and HE is equal to $\angle ADB$.
$$\begin{aligned} \tan\angle ADB &= \frac{AB}{AD} \\ \angle ADB &= \tan^{-1}\frac{AB}{AD} \\ &= \tan^{-1}\frac{62}{38} \\ &= 58.5° \end{aligned}$$

3. The key to this problem and others like it is a clear diagram that captures the information given.



(a)
$$\begin{aligned} BF^2 &= BE^2 + EF^2 \\ &= (AB^2 + AE^2) + EF^2 \\ &= 100^2 + 30^2 + 120^2 \\ BF &= \sqrt{25300} \\ &= 159\text{mm} \end{aligned}$$

(b)
$$\begin{aligned} \sin\angle FBD &= \frac{DF}{BF} \\ &= \frac{30}{159} \\ \angle FBD &= \sin^{-1}\frac{30}{159} \\ &= 10.9° \end{aligned}$$

4. (a)
$$\begin{aligned} \tan 50° &= \frac{186}{AC} \\ AC &= \frac{186}{\tan 50°} \\ &= 156\text{cm} \end{aligned}$$

(b)
$$\begin{aligned} \tan 24° &= \frac{186}{AB} \\ AB &= \frac{186}{\tan 24°} \\ &= 418\text{cm} \end{aligned}$$
$$\begin{aligned} BC &= \sqrt{AB^2 + AC^2} \\ &= \sqrt{156^2 + 418^2} \\ &= 446\text{cm} \end{aligned}$$

(c) $\tan \angle ACB = \dfrac{AB}{AC}$

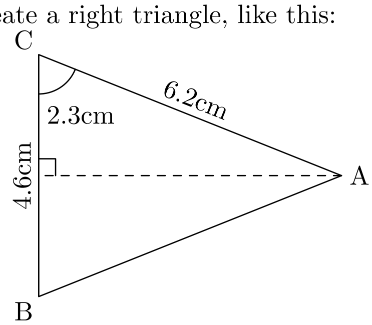
$$\angle ACB = \tan^{-1} \frac{418}{156} = 69.5°$$

5. (a) $\triangle DCA \cong \triangle DBA$ (SAS) so $\angle DCA \cong \angle DBA$ and $\angle DBA = 50°$

(b) $\angle DBA = \dfrac{DA}{AB}$

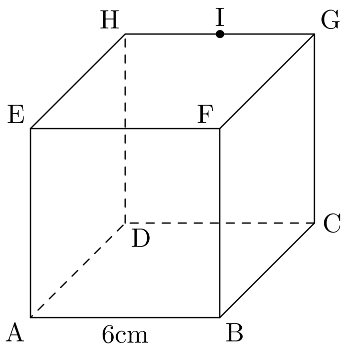
$$\begin{aligned} AB &= \frac{DA}{\tan \angle DBA} \\ &= \frac{7.4}{\tan 50°} \\ &= 6.2\text{cm} \end{aligned}$$

(c) There are a couple of ways this could be done. Since we now know all three sides of triangle ABC we could use the cosine rule to find $\angle ACB$. Alternatively, since we have an isosceles triangle, we can divide it in half to create a right triangle, like this:



$$\begin{aligned} \cos \angle ACB &= \frac{2.3}{6.2} \\ \angle ACB &= \cos^{-1} \frac{2.3}{6.2} \\ &= 68° \end{aligned}$$

6.

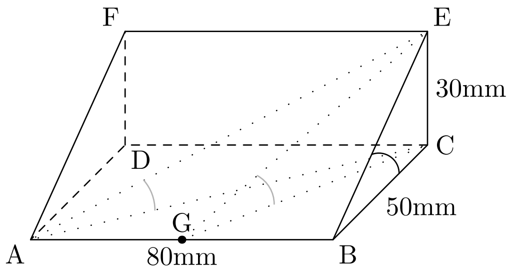


(a)
$$\begin{aligned} BI &= \sqrt{BF^2 + FI^2} \\ &= \sqrt{BF^2 + FG^2 + GI^2} \\ &= \sqrt{6^2 + 6^2 + 3^2} \\ &= 9 \end{aligned}$$

(b)
$$\begin{aligned} \cos \angle IBF &= \frac{BF}{FI} \\ &= \frac{6}{9} \\ &= \frac{2}{3} \\ \angle IBF &= \cos^{-1} \frac{2}{3} \\ &= 48° \end{aligned}$$

(c) The angle between $\triangle IAB$ and the base ABCD is the same as the angle between rectangle ABGH and the base ABCD (since the triangle and the rectangle are coplanar). This is the same as $\angle GBC$: 45°.

7.

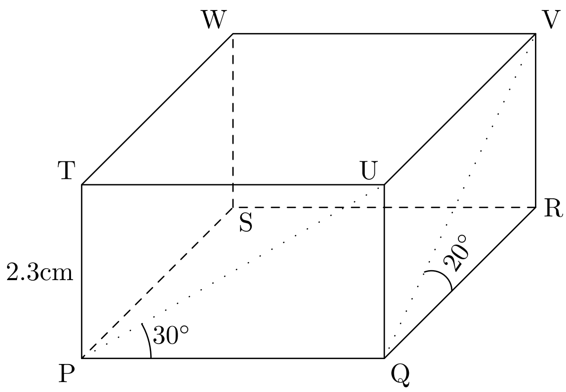


(a)
$$\begin{aligned} \tan \angle EBC &= \frac{CE}{BC} \\ \angle EBC &= \tan^{-1} \frac{30}{50} \\ &= 31° \end{aligned}$$

(b)
$$\begin{aligned} \tan \angle EGC &= \frac{CE}{GC} \\ \angle EBC &= \tan^{-1} \frac{30}{\sqrt{40^2 + 50^2}} \\ &= 25° \end{aligned}$$

(c)
$$\begin{aligned} \tan \angle EAC &= \frac{CE}{AC} \\ \angle EBC &= \tan^{-1} \frac{30}{\sqrt{80^2 + 50^2}} \\ &= 18° \end{aligned}$$

8.



(a) $\tan 30° = \dfrac{QU}{PQ}$ $\quad$ $\tan 20° = \dfrac{VR}{QR}$

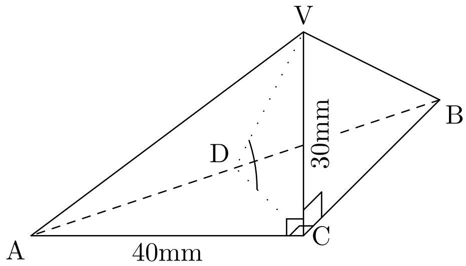
$$PQ = \frac{2.3}{\tan 30°} = 3.98\text{cm} \qquad QR = \frac{2.3}{\tan 20°} = 6.32\text{cm}$$

$$\begin{aligned} \text{Volume} &= 2.3 \times 3.98 \times 6.32 \\ &= 57.9\text{cm}^3 \end{aligned}$$

(b) $$\begin{aligned} \text{PV} &= \sqrt{\text{PQ}^2 + \text{QR}^2 + \text{RV}^2} \\ &= \sqrt{3.98^2 + 6.32^2 + 2.3^2} \\ &= 7.8\text{cm} \end{aligned}$$

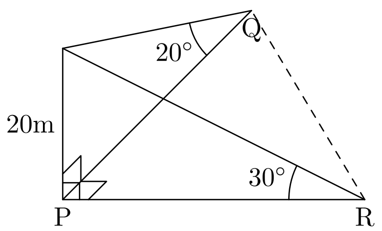
(c) $$\begin{aligned} \tan\angle\text{USW} &= \frac{\text{UW}}{\text{SW}} \\ \angle\text{USW} &= \tan^{-1}\frac{\sqrt{3.98^2+6.32^2}}{2.3} \\ &= 73^\circ \end{aligned}$$

9. The angle between plane VAB and plane ABC is equal to the angle between lines that are both perpendicular to AB. Consider point D the mid-point of AB such that VD and VC are both perpendicular to AB.



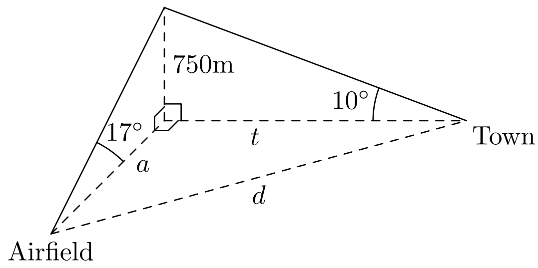
$$\begin{aligned} \sin 45^\circ &= \frac{\text{DC}}{\text{AC}} \\ \text{DC} &= 40\sin 45^\circ \\ &= 28.28\text{mm} \\ \tan\angle\text{VDC} &= \frac{\text{VC}}{\text{DC}} \\ \angle\text{VDC} &= \tan^{-1}\frac{30}{28.28} \\ &= 47^\circ \end{aligned}$$

10.



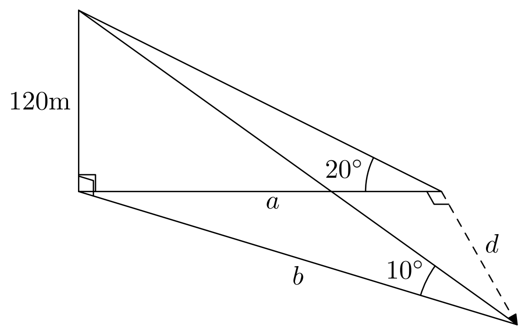
$$\begin{aligned} \text{PQ} &= \frac{20}{\tan 20^\circ} \\ &= 54.9\text{m} \\ \text{PR} &= \frac{20}{\tan 30^\circ} \\ &= 34.6\text{m} \\ \text{QR} &= \sqrt{\text{PQ}^2 + \text{PR}^2} \\ &= 65\text{m} \end{aligned}$$

11.

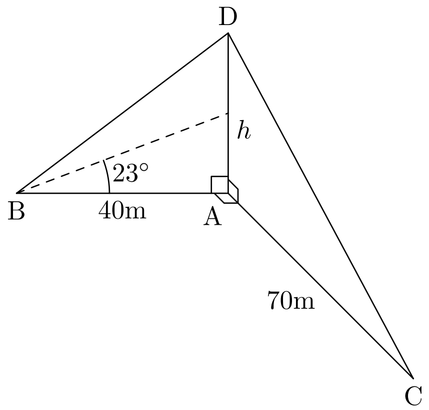


$$\begin{aligned} a &= \frac{750}{\tan 17^\circ} \\ &= 2453\text{m} \\ t &= \frac{750}{\tan 30^\circ} \\ &= 4253\text{m} \\ d &= \sqrt{a^2 + t^2} \\ &= 4910\text{m} \\ &\approx 5\text{km} \end{aligned}$$

12.



$$\begin{aligned} a &= \frac{120}{\tan 20^\circ} \\ &= 330\text{m} \\ b &= \frac{120}{\tan 30^\circ} \\ &= 681\text{m} \\ d &= \sqrt{b^2 - a^2} \\ &= 595\text{m} \\ \text{Speed} &= \frac{595}{10} \\ &= 59.5\text{m/min} \\ &= 59.5 \times 60\text{m/hr} \\ &= 3572\text{m/hr} \\ &= 3.6\text{km/hr} \end{aligned}$$

13. 

(a) $$\frac{h}{2} = 40\tan 23°$$
$$h = 33.96\text{m}$$
$$\angle\text{ABD} = \tan^{-1}\frac{h}{40} = 40°$$

(b) $$\angle\text{ACD} = \tan^{-1}\frac{h}{70} = 26°$$

14. $$\text{BD} = h$$
$$\text{AB} = \frac{h}{\sin 28°}$$
$$\cos 35° = \frac{\text{AB}}{\text{AC}}$$
$$\text{AC} = \frac{\text{AB}}{\cos 35°} = \frac{\frac{h}{\sin 28°}}{\cos 35°} = \frac{h}{\sin 28° \cos 35°}$$
$$\sin\theta = \frac{h}{\text{AC}} = \frac{h}{1} \times \frac{\sin 28° \cos 35°}{h} = \sin 28° \cos 35°$$
$$\theta = \sin^{-1}(\sin 28° \cos 35°) = 23°$$

Exercise 2C

1. (a) $$c^2 = a^2 + b^2 - 2ab\cos C$$
$$10.2^2 = x^2 + 6.9^2 - 2 \times x \times 6.9 \times \cos 50°$$
$$x = -4.29 \text{ or } x = 13.16$$
Reject the negative solution and round to 1d.p.: $x = 13.2$cm.

(b) $$\frac{\sin A}{a} = \frac{\sin C}{c}$$
$$\sin A = \frac{a\sin C}{c}$$
$$A = \sin^{-1}\frac{a\sin C}{c} = \sin^{-1}\frac{6.9\sin 50°}{10.2} = 31.2°$$
$$\text{or } A = 180 - 31.2 = 148.8$$
Reject the obtuse solution since it results in an internal angle sum greater than 180°.

$$B = 180 - A - C = 180 - 50 - 31.2 = 98.8°$$
$$\frac{x}{\sin 98.8} = \frac{10.2}{\sin 50}$$
$$x = \frac{10.2\sin 98.8}{\sin 50} = 13.2\text{cm}$$

2. $$\frac{\sin x}{11.2} = \frac{\sin 50°}{12.1}$$
$$x = \sin^{-1}\frac{11.2\sin 50}{12.1} = 45°$$
No need to consider the obtuse solution since the opposite side is not the longest in the triangle (x must be less than 50°).

3. $$x^2 = 6.8^2 + 14.3^2 - 2 \times 6.8 \times 14.3 \times \cos 20°$$
$$x = \sqrt{6.8^2 + 14.3^2 - 2 \times 6.8 \times 14.3 \times \cos 20°} = 8.2\text{cm}$$

4. $19.7^2 = 9.8^2 + 14.3^2 - 2 \times 9.8 \times 14.3 \times \cos x$

$$\cos x = \frac{9.8^2 + 14.3^2 - 19.7^2}{2 \times 9.8 \times 14.3}$$

$$x = \cos^{-1} \frac{9.8^2 + 14.3^2 - 19.7^2}{2 \times 9.8 \times 14.3}$$

$$= 108°$$

5. $$\frac{x}{\sin(180 - 105 - 25)} = \frac{11.8}{\sin 105}$$

$$x = \frac{11.8 \sin 50}{\sin 105}$$

$$= 9.4\text{cm}$$

6. $$\frac{\sin x}{7.2} = \frac{\sin 40°}{4.8}$$

$$x = \sin^{-1} \frac{7.2 \sin 40°}{4.8}$$

$$= 75° \quad \text{or } x = 180 - 75$$

$$= 105°$$

7. $$c^2 = a^2 + b^2 - 2ab \cos C$$

$$11.8^2 = x^2 + 8.7^2 - 2 \times x \times 8.7 \times \cos 80°$$

$$x = 9.6$$

(rejecting the negative solution)

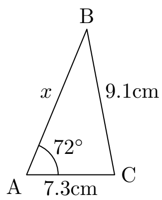
8. The smallest angle is opposite the shortest side, so

$$27^2 = 33^3 + 55^2 - 2 \times 33 \times 55 \times \cos \theta$$

$$\theta = \cos^{-1} \frac{33^2 + 55^2 - 27^2}{2 \times 33 \times 55}$$

$$= 21°$$

9. $$a^2 = b^2 + c^2 - 2bc \cos A$$



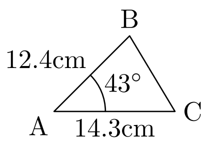
$$9.1^2 = 7.3^2 + x^2 - 2 \times 7.3x \cos 72°$$

$$x = 8.1$$

(rejecting the negative solution)

AB=8.1cm

10.



$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$a = \sqrt{12.4^2 + 14.3^2 - 2 \times 12.4 \times 14.3 \cos 43°}$$

$$a = 9.9\text{cm}$$

$$\frac{\sin C}{12.4} = \frac{\sin 43°}{a}$$

$$C = \sin^{-1} \frac{12.4 \sin 43°}{9.9}$$

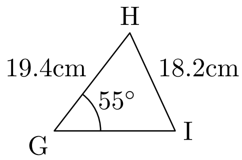
$$= 58°$$

(Cannot be obtuse because c is not the longest side.)

$$B = 180 - 43 - 58$$

$$= 79°$$

11.



$$\frac{\sin I}{19.4} = \frac{\sin 55°}{18.2}$$

$$I = \sin^{-1} \frac{19.4 \sin 55°}{18.2}$$

$$= 61° \quad \text{or} \quad 119°$$

$$H = 180 - 55 - 61 \quad \text{or} \quad 180 - 55 - 119$$

$$= 64° \quad \quad = 6°$$

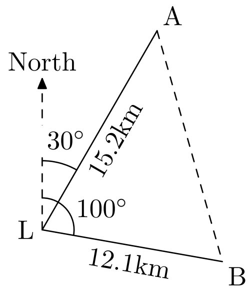
$$\frac{h}{\sin H} = \frac{g}{\sin G}$$

$$h = \frac{g \sin H}{\sin G}$$

$$= \frac{18.2 \sin 64°}{\sin 55°} \quad \text{or} \quad \frac{18.2 \sin 6°}{\sin 55°}$$

$$= 20.0\text{cm} \quad \quad = 2.3\text{cm}$$

12.



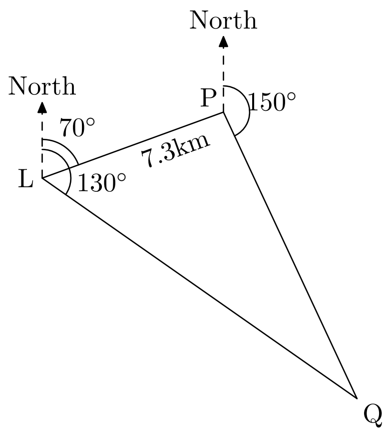
$$\angle \text{ALB} = 100 - 30$$

$$= 70°$$

$$\text{AB} = \sqrt{15.2^2 + 12.1^2 - 2 \times 15.2 \times 12.1 \cos 70°}$$

$$= 15.9\text{km}$$

13.



$$\angle \text{PQL} = 150 - 130$$

$$= 20°$$

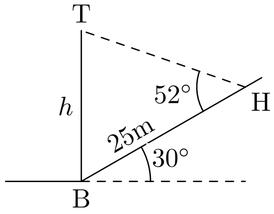
$$\angle \text{PLQ} = 130 - 70$$

$$= 60°$$

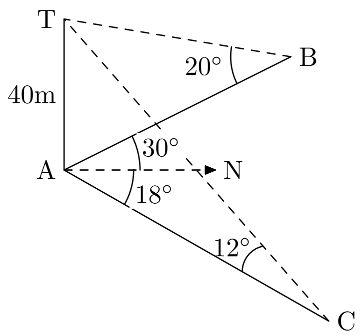
$$\angle \text{LPQ} = 180 - 20 - 60$$

$$= 100°$$

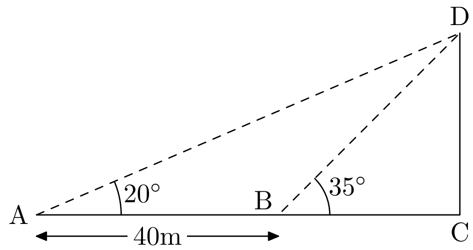
$$\begin{aligned}\frac{\text{LQ}}{\sin\angle\text{LPQ}} &= \frac{\text{LP}}{\sin\angle\text{PQL}}\\ \text{LQ} &= \frac{\text{LP}\sin\angle\text{LPQ}}{\sin\angle\text{PQL}}\\ &= \frac{7.3\sin 100^\circ}{\sin 20^\circ}\\ &= 21.0\text{km}\end{aligned}$$

14. 

$$\begin{aligned}\angle\text{HBT} &= 90-30\\ &= 60^\circ\\ \angle\text{BTH} &= 180-60-52\\ &= 68^\circ\\ \frac{h}{\sin 52^\circ} &= \frac{25}{\sin 68^\circ}\\ h &= \frac{25\sin 52^\circ}{\sin 68^\circ}\\ &= 21\text{m}\end{aligned}$$

15. 

$$\begin{aligned}\tan 20^\circ &= \frac{40}{\text{AB}}\\ \text{AB} &= \frac{40}{\tan 20^\circ}\\ &= 109.9\text{m}\\ \tan 12^\circ &= \frac{40}{\text{AC}}\\ \text{AC} &= \frac{40}{\tan 12^\circ}\\ &= 188.2\text{m}\\ \angle\text{BAC} &= 30+18\\ &= 48^\circ\\ \text{BC}^2 &= \text{AB}^2+\text{AC}^2-2\text{AB}\times\text{AC}\cos\angle\text{BAC}\\ \text{BC} &= \sqrt{109.9^2+188.2^2-2\times109.9\times188.2\cos 48^\circ}\\ &= 141\text{m}\end{aligned}$$

16. 

$$\begin{aligned}\angle\text{ADB} &= 35-20\\ &= 15^\circ\\ \frac{\text{BD}}{\sin 20^\circ} &= \frac{40}{\sin 15^\circ}\\ \text{BD} &= \frac{40\sin 20^\circ}{\sin 15^\circ}\\ &= 52.9\text{m}\\ \sin\angle\text{DBC} &= \frac{\text{DC}}{\text{BD}}\\ \text{DC} &= \text{BD}\sin\angle\text{DBC}\\ &= 52.9\sin 35\\ &= 30\text{m}\end{aligned}$$

17. There are a couple of ways you could approach this problem. You could use the cosine rule to determine an angle, then use the formula Area= $\frac{1}{2}ab\sin C$. Alternatively you could use Heron's formula:

$$A=\sqrt{s(s-a)(s-b)(s-c)}$$

where $s=\frac{a+b+c}{2}$ and determine the area without resort to trigonometry at all. I'll use trigonometry for the first block, and Heron's formula for the second.
First block—I'll start by finding the largest angle:

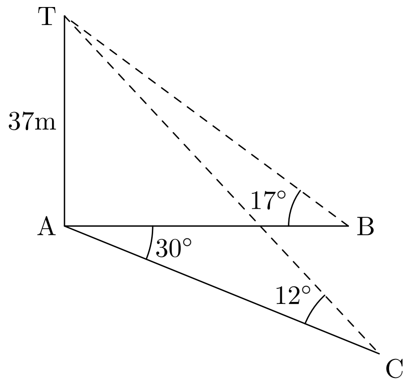
$$\begin{aligned}\theta &= \cos^{-1}\frac{25^2+48^2-53^2}{2\times25\times48}\\ &= 87.1^\circ\\ \text{Area} &= \frac{1}{2}ab\sin\theta\\ &= \frac{1}{2}\times25\times48\sin 87.1\\ &= 599.2\text{m}^2\end{aligned}$$

Second block:

$$\begin{aligned}s &= \frac{33+38+45}{2}\\ &= 58\\ \text{Area} &= \sqrt{58(58-33)(58-38)(58-45)}\\ &= 614.0\text{m}^2\end{aligned}$$

The second block is larger by 15m^2.

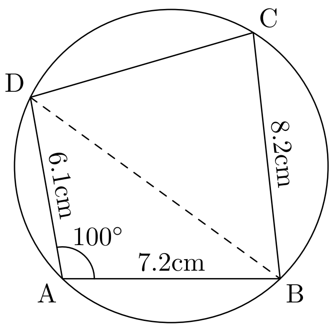
18.



$$\tan 17^\circ = \frac{37}{\text{AB}}$$
$$\text{AB} = \frac{37}{\tan 17^\circ} = 121.0\text{m}$$

$$\tan 12^\circ = \frac{37}{\text{AC}}$$
$$\text{AC} = \frac{37}{\tan 12^\circ} = 174.0\text{m}$$
$$\text{BC}^2 = \text{AB}^2 + \text{AC}^2 - 2 \times \text{AB} \times \text{AC} \cos 30^\circ$$
$$\text{BC} = \sqrt{121.0^2 + 174.0^2 - 2 \times 121.0 \times 174.0 \cos 30^\circ} = 92.0\text{m}$$

19.



(a) $\angle\text{BCD} = 180 - 100 = 80^\circ$

(b) $\text{BD}^2 = 6.1^2 + 7.2^2 - 2 \times 6.1 \times 7.2 \cos 100^\circ = 104.3$

$\text{BD} = 10.2\text{cm}$

$$\frac{\sin\angle\text{ADB}}{7.2} = \frac{\sin 100^\circ}{10.2}$$
$$\angle\text{ADB} = \sin^{-1}\frac{7.2\sin 100^\circ}{10.2} = 44.0^\circ$$
$$\frac{\sin\angle\text{CDB}}{8.2} = \frac{\sin 80^\circ}{10.2}$$
$$\angle\text{CDB} = \sin^{-1}\frac{8.2\sin 100^\circ}{10.2} = 52.3^\circ$$
$$\angle\text{ADC} = 44.0 + 52.3 = 96^\circ$$

(c) $\text{BD}^2 = \text{BC}^2 + \text{CD}^2 - 2 \times \text{BC} \times \text{CD} \cos 80^\circ$

$104.3 = 8.2^2 + \text{CD}^2 - 2 \times 8.2 \times \text{CD} \cos 80^\circ$

$\text{CD} = 7.7\text{cm}$

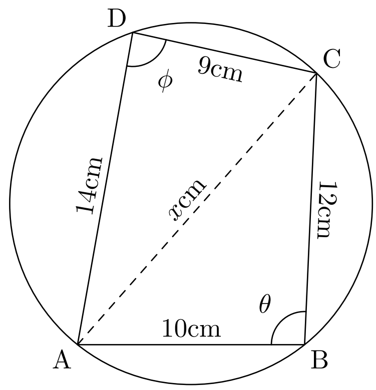
$P = 7.2 + 8.2 + 7.7 + 6.1 = 29.2\text{cm}$

(d) $A_{\triangle ABD} = \frac{1}{2} \times 6.1 \times 7.2 \times \sin 100^\circ = 21.6\text{cm}^2$

$A_{\triangle CBD} = \frac{1}{2} \times 8.2 \times 7.7 \times \sin 80^\circ = 31.1\text{cm}^2$

$A_{ABCD} = 21.6 + 31.1 = 52.7\text{cm}^2$

20.



(a) $x^2 = 10^2 + 12^2 - 2 \times 10 \times 12 \cos\theta = 100 + 144 - 240\cos\theta = 244 - 240\cos\theta$

(b) $x^2 = 14^2 + 9^2 - 2 \times 14 \times 9 \cos\phi = 196 + 81 - 252\cos\phi = 277 - 252\cos\phi$

(c)
$$\phi = 180^\circ - \theta$$
$$\cos\phi = \cos(180^\circ - \theta) = -\cos\theta$$
$$244 - 240\cos\theta = 277 - 252\cos\phi = 277 + 252\cos\theta$$
$$-240\cos\theta = 33 + 252\cos\theta$$
$$-492\cos\theta = 33$$
$$\cos\theta = -\frac{33}{492}$$
$$\theta = 94^\circ$$

Exercise 2D

Questions 1–15 are single step problems. No worked solutions necessary.

Note: My exact values are given with rational denominators. I write $\frac{\sqrt{2}}{2}$ rather than $\frac{1}{\sqrt{2}}$. Your answers may appear different without being wrong.

16. $120°$ makes an angle of $60°$ with the x-axis and is in quadrant II (where sine is positive) so $\sin 120° = \sin 60° = \frac{\sqrt{3}}{2}$.
17. $135°$ makes an angle of $45°$ with the x-axis and is in quadrant II (where cosine is negative) so $\cos 135° = -\cos 45° = -\frac{\sqrt{2}}{2}$.
18. $150°$ makes an angle of $30°$ with the x-axis and is in quadrant II (where cosine is negative) so $\cos 150° = -\cos 30° = -\frac{\sqrt{3}}{2}$.
19. $120°$ makes an angle of $60°$ with the x-axis and is in quadrant II (where cosine is negative) so $\cos 120° = -\cos 60° = -\frac{1}{2}$.
20. $180°$ makes an angle of $0°$ with the x-axis and is on the negative x-axis (where cosine is negative) so $\cos 180° = -\cos 0° = -1$.
21. $135°$ makes an angle of $45°$ with the x-axis and is in quadrant II (where tangent is negative) so $\tan 135° = -\tan 45° = -1$.
22. $120°$ makes an angle of $60°$ with the x-axis and is in quadrant II (where tangent is negative) so $\tan 120° = -\tan 60° = -\sqrt{3}$.
23. $150°$ makes an angle of $30°$ with the x-axis and is in quadrant II (where tangent is negative) so $\tan 150° = -\tan 30° = \frac{\sqrt{3}}{3}$.
24. $180°$ lies on the negative x-axis (where tangent is zero) so $\tan 180° = 0$.
25. $180°$ lies on the negative x-axis (where sine is zero) so $\sin 180° = 0$.
26. $150°$ makes an angle of $30°$ with the x-axis and is in quadrant II (where sine is positive) so $\sin 150° = \sin 30° = \frac{1}{2}$.
27. $135°$ makes an angle of $45°$ with the x-axis and is in quadrant II (where sine is positive) so $\sin 135° = \sin 45° = \frac{\sqrt{2}}{2}$.
28. $\sqrt{20} = \sqrt{4 \times 5} = \sqrt{4}\sqrt{5} = 2\sqrt{5}$
29. $\sqrt{45} = \sqrt{9 \times 5} = \sqrt{9}\sqrt{5} = 3\sqrt{5}$
30. $\sqrt{32} = \sqrt{16 \times 2} = \sqrt{16}\sqrt{2} = 4\sqrt{2}$
31. $\sqrt{72} = \sqrt{36 \times 2} = \sqrt{36}\sqrt{2} = 6\sqrt{2}$
32. $\sqrt{50} = \sqrt{25 \times 2} = \sqrt{25}\sqrt{2} = 5\sqrt{2}$
33. $\sqrt{200} = \sqrt{100 \times 2} = \sqrt{100}\sqrt{2} = 10\sqrt{2}$
34. $\sqrt{2} \times \sqrt{3} = \sqrt{2 \times 3} = \sqrt{6}$
35. $\sqrt{5} \times \sqrt{3} = \sqrt{5 \times 3} = \sqrt{15}$
36. $\sqrt{5} \times \sqrt{5} = (\sqrt{5})^2 = 5$
37. $\sqrt{15} \times \sqrt{3} = \sqrt{15 \times 3} = \sqrt{9 \times 5} = \sqrt{9}\sqrt{5} = 3\sqrt{5}$
38. $\sqrt{8} \times \sqrt{6} = \sqrt{8 \times 6} = \sqrt{16 \times 3} = \sqrt{16}\sqrt{3} = 4\sqrt{3}$
39. $3\sqrt{2} \times 4\sqrt{2} = 12\sqrt{2}\sqrt{2} = 12(\sqrt{2})^2 = 12 \times 2 = 24$
40. $(5\sqrt{2})(3\sqrt{8}) = 15\sqrt{2}\sqrt{8} = 15\sqrt{2 \times 8} = 15\sqrt{16} = 15 \times 4 = 60$
41. $(6\sqrt{3})(\sqrt{12}) = 6\sqrt{3 \times 12} = 6\sqrt{36} = 6 \times 6 = 36$
42. $(3\sqrt{5})(7\sqrt{2}) = 21\sqrt{5 \times 2} = 21\sqrt{10}$
43. $(5\sqrt{2}) \div (\sqrt{8}) = 5\sqrt{2} \div \sqrt{4 \times 2} = 5\sqrt{2} \div (2\sqrt{2}) = 5 \div 2 = 2.5$
44. $(5\sqrt{3})^2 = 5^2 \times (\sqrt{3})^2 = 25 \times 3 = 75$
45. $(3\sqrt{2})^2 = 3^2 \times (\sqrt{2})^2 = 9 \times 2 = 18$
46. $\frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$
47. $\frac{1}{\sqrt{3}} = \frac{1}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$
48. $\frac{1}{\sqrt{5}} = \frac{1}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{\sqrt{5}}{5}$
49. $\frac{3}{\sqrt{2}} = \frac{3}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{3\sqrt{2}}{2}$
50. $\frac{2}{\sqrt{7}} = \frac{2}{\sqrt{7}} \times \frac{\sqrt{7}}{\sqrt{7}} = \frac{2\sqrt{7}}{7}$
51. $\frac{6}{\sqrt{3}} = \frac{6}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{6\sqrt{3}}{3} = 2\sqrt{3}$
52. $\frac{1}{3+\sqrt{5}} = \frac{1}{3+\sqrt{5}} \times \frac{3-\sqrt{5}}{3-\sqrt{5}} = \frac{3-\sqrt{5}}{9-5} = \frac{3-\sqrt{5}}{4}$
53. $\frac{1}{3-\sqrt{2}} = \frac{1}{3-\sqrt{2}} \times \frac{3+\sqrt{2}}{3+\sqrt{2}} = \frac{3+\sqrt{2}}{9-2} = \frac{3+\sqrt{2}}{7}$
54. $\frac{1}{3+\sqrt{2}} = \frac{1}{3+\sqrt{2}} \times \frac{3-\sqrt{2}}{3-\sqrt{2}} = \frac{3-\sqrt{2}}{9-2} = \frac{3-\sqrt{2}}{7}$
55. $\frac{2}{\sqrt{3}+\sqrt{2}} = \frac{2}{\sqrt{3}+\sqrt{2}} \times \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}} = \frac{2(\sqrt{3}-\sqrt{2})}{3-2} = 2\sqrt{3} - 2\sqrt{2}$
56. $\frac{3}{\sqrt{3}-\sqrt{2}} = \frac{3}{\sqrt{3}-\sqrt{2}} \times \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}} = \frac{3(\sqrt{3}+\sqrt{2})}{3-2} = 3\sqrt{3} + 3\sqrt{2}$
57. $\frac{6}{\sqrt{5}+\sqrt{2}} = \frac{6}{\sqrt{5}+\sqrt{2}} \times \frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}-\sqrt{2}} = \frac{6(\sqrt{5}-\sqrt{2})}{5-2} = \frac{6(\sqrt{5}-\sqrt{2})}{3} = 2\sqrt{5} - 2\sqrt{2}$

58. $$\begin{aligned}\sin 60^\circ &= \frac{9}{x}\\ \frac{\sqrt{3}}{2} &= \frac{9}{x}\\ \sqrt{3}x &= 18\\ x &= \frac{18}{\sqrt{3}}\\ &= \frac{18}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}\\ &= \frac{18\sqrt{3}}{3}\\ &= 6\sqrt{3}\end{aligned}$$

59. $$\begin{aligned}x^2 + 3^2 &= 7^2\\ x^2 + 9 &= 49\\ x^2 &= 40\\ x &= \sqrt{40}\\ &= \sqrt{4 \times 10}\\ &= \sqrt{4} \times \sqrt{10}\\ &= 2\sqrt{10}\end{aligned}$$

60. Label the vertical in the diagram as y, then

$$\begin{aligned}\sin 45^\circ &= \frac{y}{10}\\ \frac{\sqrt{2}}{2} &= \frac{y}{10}\\ y &= 5\sqrt{2}\\ \sin 60^\circ &= \frac{x}{y}\\ \frac{\sqrt{3}}{2} &= \frac{x}{5\sqrt{2}}\\ x &= \frac{5\sqrt{2}}{1} \times \frac{\sqrt{3}}{2}\\ &= \frac{5\sqrt{3}\sqrt{2}}{2}\\ &= \frac{5\sqrt{6}}{2}\end{aligned}$$

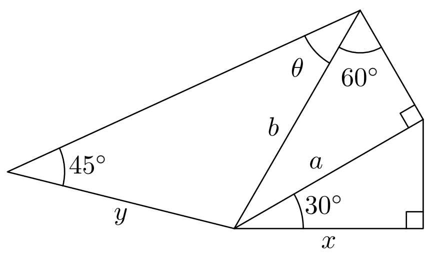
61. Use the cosine rule:

$$\begin{aligned}x^2 &= 4^2 + (2\sqrt{3})^2 - 2 \times 4 \times 2\sqrt{3} \times \cos 150^\circ\\ &= 16 + 2^2 \times (\sqrt{3})^2 - 16\sqrt{3} \times (-\cos 30^\circ)\\ &= 16 + 4 \times 3 - 16\sqrt{3} \times \left(-\frac{\sqrt{3}}{2}\right)\\ &= 16 + 12 + \frac{16\sqrt{3} \times \sqrt{3}}{2}\\ &= 28 + 8 \times 3\\ &= 52\\ x &= \sqrt{52}\\ &= \sqrt{4 \times 13}\\ &= 2\sqrt{13}\end{aligned}$$

62. Label the diagonal in the diagram as y, then

$$\begin{aligned}\frac{y}{\sin 60^\circ} &= \frac{10}{\sin 45^\circ}\\ y &= \frac{10 \sin 60^\circ}{\sin 45^\circ}\\ &= 10 \times \frac{\sqrt{3}}{2} \div \frac{1}{\sqrt{2}}\\ &= \frac{5\sqrt{3}}{1} \times \frac{\sqrt{2}}{1}\\ &= 5\sqrt{3}\sqrt{2}\\ &= 5\sqrt{6}\\ \tan 30^\circ &= \frac{x}{y}\\ x &= y \tan 30^\circ\\ &= 5\sqrt{6} \times \frac{1}{\sqrt{3}}\\ &= \frac{5\sqrt{3}\sqrt{2}}{\sqrt{3}}\\ &= 5\sqrt{2}\end{aligned}$$

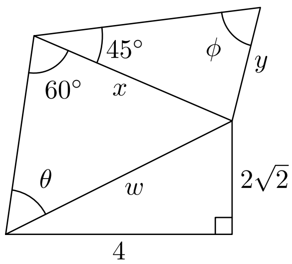
63.



$$\begin{aligned}\cos 30^\circ &= \frac{x}{a}\\ \frac{\sqrt{3}}{2} &= \frac{x}{a}\\ \sqrt{3}a &= 2x\\ a &= \frac{2x}{\sqrt{3}}\\ \sin 60^\circ &= \frac{a}{b}\\ \frac{\sqrt{3}}{2} &= \frac{a}{b}\\ \sqrt{3}b &= 2a\\ &= 2 \times \frac{2x}{\sqrt{3}}\\ &= \frac{4x}{\sqrt{3}}\\ b &= \frac{4x}{\sqrt{3}} \times \frac{1}{\sqrt{3}}\\ &= \frac{4x}{3}\end{aligned}$$

$$\begin{aligned}\frac{y}{\sin\theta} &= \frac{b}{\sin 45°}\\ y &= b\times\sin\theta\div\sin 45°\\ &= b\sin\theta\div\frac{1}{\sqrt{2}}\\ &= b\sin\theta\times\frac{\sqrt{2}}{1}\\ &= \sqrt{2}b\sin\theta\\ &= \sqrt{2}\times\frac{4x}{3}\times\sin\theta\\ &= \frac{4\sqrt{2}x\sin\theta}{3}\end{aligned}$$

□

64. 

$$\begin{aligned}w^2 &= 4^2+(2\sqrt{2})^2\\ &= 16+4\times 2\\ &= 24\\ w &= \sqrt{24}\\ &= \sqrt{4\times 6}\\ &= 2\sqrt{6}\end{aligned}$$

$$\begin{aligned}\frac{x}{\sin\theta} &= \frac{w}{\sin 60°}\\ x &= \frac{w\sin\theta}{\sin 60°}\\ &= \frac{2\sqrt{6}\sin\theta}{\frac{\sqrt{3}}{2}}\\ &= \frac{2\sqrt{6}\sin\theta}{1}\times\frac{2}{\sqrt{3}}\\ &= \frac{4\sqrt{3}\sqrt{2}\sin\theta}{\sqrt{3}}\\ &= 4\sqrt{2}\sin\theta\\ \frac{y}{\sin 45°} &= \frac{x}{\sin\phi}\\ y &= \frac{x\sin 45°}{\sin\phi}\\ &= \frac{x\times\frac{1}{\sqrt{2}}}{\sin\phi}\\ &= \frac{4\sqrt{2}\sin\theta\times\frac{1}{\sqrt{2}}}{\sin\phi}\\ &= \frac{4\sin\theta}{\sin\phi}\end{aligned}$$

□

Exercise 2E

1. $43-19=24°$
$$\begin{aligned}d &= \frac{24}{360}\times 2\pi\times 6350\\ &= 2660\text{km}\end{aligned}$$

2. $32-21=11°$
$$\begin{aligned}d &= \frac{11}{360}\times 2\pi\times 6350\\ &= 1219\text{km}\end{aligned}$$

3. $39-(-32)=71°$
$$\begin{aligned}d &= \frac{71}{360}\times 2\pi\times 6350\\ &= 7869\text{km}\end{aligned}$$

4. $51.5-5=46.5°$
$$\begin{aligned}d &= \frac{46.5}{360}\times 2\pi\times 6350\\ &= 5154\text{km}\end{aligned}$$

5. $41-4=37°$
$$\begin{aligned}d &= \frac{37}{360}\times 2\pi\times 6350\\ &= 4101\text{km}\end{aligned}$$

6. $134-114=20°$
$$\begin{aligned}d &= \frac{20}{360}\times 2\pi\times 6350\cos 25°\\ &= 2009\text{km}\end{aligned}$$

7. $119 - 77 = 42°$
$$\begin{aligned} d &= \frac{42}{360} \times 2\pi \times 6350 \cos 39° \\ &= 3617\text{km} \end{aligned}$$

8. $105 - 75 = 30°$
$$\begin{aligned} d &= \frac{30}{360} \times 2\pi \times 6350 \cos 40° \\ &= 2547\text{km} \end{aligned}$$

9. $122 - 117 = 5°$
$$\begin{aligned} d &= \frac{5}{360} \times 2\pi \times 6350 \cos 34° \\ &= 459\text{km} \end{aligned}$$

10. $175 - (-73) = 248°$
Longitude difference is greater than 180° so it is shorter to go the other way and cross the date line.
$360 - 248 = 112°$
$$\begin{aligned} d &= \frac{112}{360} \times 2\pi \times 6350 \cos 40° \\ &= 9509\text{km} \end{aligned}$$

11.
$$\begin{aligned} \frac{\theta}{360} &= \frac{555}{2\pi \times 6350} \\ \theta &= \frac{555}{2\pi \times 6350} \times 360 \\ &= 5° \\ \text{latitude} &= 29 + 5 \\ &= 34°\text{S} \end{aligned}$$
Augusta: 34°S, 115°E

12.
$$\begin{aligned} \frac{\theta}{360} &= \frac{3300}{2\pi \times 6350 \cos 34°} \\ \theta &= \frac{3300}{2\pi \times 6350 \cos 34°} \times 360 \\ &= 36° \\ \text{longitude} &= 115 + 36 \\ &= 151°\text{E} \end{aligned}$$
Sydney: 34°S, 151°E

13.
$$\begin{aligned} \frac{\theta}{360} &= \frac{7870}{2\pi \times 6350} \\ \theta &= \frac{7870}{2\pi \times 6350} \times 360 \\ &= 71° \\ \text{latitude} &= 71 - 36 \\ &= 35°\text{S} \end{aligned}$$
Adelaide: 35°S, 138°E

14.
$$\begin{aligned} \frac{\theta}{360} &= \frac{9600}{2\pi \times 6350 \cos 35°} \\ \theta &= \frac{9600}{2\pi \times 6350 \cos 35°} \times 360 \\ &= 106° \\ \text{longitude} &= 135 + 106 \\ &= 241°\text{E} \\ &= 360 - 241 \\ &= 119°\text{W} \end{aligned}$$
Bakersfield: 35°N, 119°W

15.
$$\begin{aligned} \frac{\theta}{360} &= \frac{820}{2\pi \times 6350 \cos 35°} \\ \theta &= \frac{820}{2\pi \times 6350 \cos 35°} \times 360 \\ &= 9° \\ \text{longitude} &= 135 - 9 \\ &= 126°\text{W} \\ \frac{\alpha}{360} &= \frac{2000}{2\pi \times 6350} \\ \alpha &= \frac{2000}{2\pi \times 6350} \times 360 \\ &= 18° \\ \text{latitude} &= 35 + 18 \\ &= 53°\text{S} \end{aligned}$$
New position: 53°S, 126°W

If the ship first heads south, the new latitude remains 53°S.
$$\begin{aligned} \frac{\theta}{360} &= \frac{820}{2\pi \times 6350 \cos 53°} \\ \theta &= \frac{820}{2\pi \times 6350 \cos 53°} \times 360 \\ &= 12° \\ \text{longitude} &= 135 - 12 \\ &= 123°\text{W} \end{aligned}$$
New position: 53°S, 123°W

16. First find the length of the chord LS from Los Angeles to Shimoneski through the earth using the angle subtended at the middle of the latitude circle:
$$\begin{aligned} r &= 6350 \cos 34° \\ &= 5264\text{km} \\ \theta &= 360 - (131 + 118) \\ &= 111° \\ \sin \frac{\theta}{2} &= \frac{0.5\text{LS}}{r} \\ 0.5\text{LS} &= 5264 \sin 55.5° \\ \text{LS} &= 2 \times 5264 \sin 55.5° \\ &= 8677\text{km} \end{aligned}$$

Now consider the angle that same chord subtends at the centre of the earth (i.e. the centre of the great circle passing through the two points). Let's call this angle α.
$$\begin{aligned} \sin \frac{\alpha}{2} &= \frac{0.5\text{LS}}{R} \\ &= \frac{0.5 \times 8677}{6350} \\ \frac{\alpha}{2} &= 43° \\ \alpha &= 86° \end{aligned}$$

Now use this angle to determine the arc length along this great circle:
$$\begin{aligned} d &= \frac{86}{360} \times 2\pi \times 6350 \\ &= 9553\text{km} \end{aligned}$$

Miscellaneous Exercise 2

1. See the answer in Sadler.

2. (a) $\tan 20° = \dfrac{15}{\text{AC}}$

$$\text{AC} = \frac{15}{\tan 20°} = 41.2\text{m}$$

(b) $\tan 30° = \dfrac{15}{\text{AB}}$

$$\text{AB} = \frac{15}{\tan 30°} = 26.0\text{m}$$

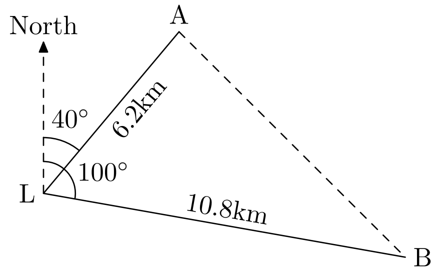
(c) $\text{BC}^2 = \text{AC}^2 + \text{AB}^2$

$$\text{BC} = \sqrt{41.2^2 + 26.0^2} = 48.7\text{m}$$

(d) $\tan \angle\text{ABC} = \dfrac{\text{AC}}{\text{AB}}$

$$\angle\text{ABC} = \tan^{-1}\frac{41.2}{26.0} = 58°$$

$$\text{bearing} = 270 + 58 = 328°$$

3. 

$$\text{AB} = \sqrt{6.2^2 + 10.8^2 - 2 \times 6.2 \times 10.8 \times \cos 60°} = 9.4\text{km}$$

$$\frac{\sin\angle\text{LBA}}{6.2} = \frac{\sin 60°}{9.4}$$

$$\angle\text{LBA} = \sin^{-1}\frac{6.2\sin 60°}{9.4} = 35°$$

$$\text{bearing} = (100 + 180) + 35 = 315°$$

4. Let l be the length of the ladder.

$$\cos 75° = \frac{a}{l}$$

$$l = \frac{a}{\cos 75°}$$

$$\cos\theta = \frac{\frac{5a}{4}}{l} = \frac{5a}{4} \times \frac{1}{l} = \frac{5a}{4} \times \frac{\cos 75°}{a} = \frac{5\cos 75°}{4}$$

$$\theta = \cos^{-1}\frac{5\cos 75°}{4} = 71°$$

5.
$$\frac{3-\sqrt{6}}{5+2\sqrt{6}} = \frac{3-\sqrt{6}}{5+2\sqrt{6}} \times \frac{5-2\sqrt{6}}{5-2\sqrt{6}} = \frac{(3-\sqrt{6})(5-2\sqrt{6})}{(5+2\sqrt{6})(5-2\sqrt{6})} = \frac{15-6\sqrt{6}-5\sqrt{6}+12}{25-24} = \frac{27-11\sqrt{6}}{1} = 27-11\sqrt{6}$$

6. (a) Read the question as "distance from 3 is less than distance from -11". The midpoint between -11 and 3 is -4, so the solution is $x > -4$.

(b) Read the question as "distance from 0 is less than distance from 6". The midpoint between 0 and 6 is 3, so the solution is $x < 3$.

(c) First solve the equation $|3x - 17| = |x - 3|$

$$3x - 17 = x - 3 \quad\text{or}\quad 3x - 17 = -(x-3)$$
$$2x = 14 \qquad 3x - 17 = -x + 3$$
$$x = 7 \qquad 4x = 20$$
$$x = 5$$

Now test a value for x, say $x = 6$, to determine whether the inequality holds at that point.

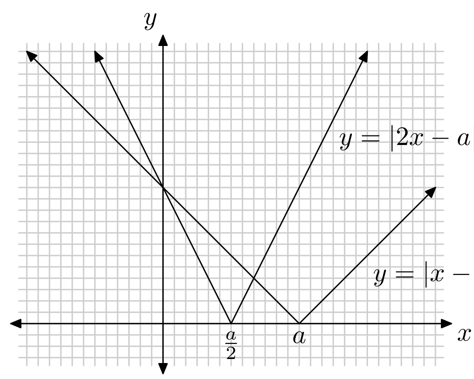
Is it true that $|3(6) - 17| \geq |(6) - 3|$
$1 \ngeq 3$: no.

Conclude that the solution lies outside the interval 5–7:

$$\{x \in \mathbb{R} : x \leq 5\} \cup \{x \in \mathbb{R} : x \geq 7\}$$

(d) This is the complementary case to the previous question, so it will have the complementary solution:

$$\{x \in \mathbb{R} : 5 < x < 7\}$$

7. 

From the graph it appears that $|2x - a| \le |x - a|$ is true for $0 \le x \le \frac{2a}{3}$. (You should confirm that these are the interval endpoints by substitution.)

8. (a)
$$\begin{aligned} \text{AH} &= \sqrt{\text{AG}^2 + \text{GH}^2} \\ &= \sqrt{4^2 + \left(\frac{12-6}{2}\right)^2} \\ &= 5\text{m} \end{aligned}$$
(b)
$$\begin{aligned} \text{EH} &= \sqrt{\text{AE}^2 - \text{AH}^2} \\ &= \sqrt{8^2 - 5^2} \\ &= \sqrt{39}\text{m} \\ &\approx 6.2\text{m} \end{aligned}$$
(c)
$$\begin{aligned} \cos\angle\text{EAH} &= \frac{\text{AH}}{\text{AE}} \\ &= \frac{5}{8} \\ \angle\text{EAH} &= 51^\circ \end{aligned}$$
(d)
$$\begin{aligned} \tan\angle\text{EGH} &= \frac{\text{EH}}{\text{GH}} \\ &= \frac{6.2}{3} \\ \angle\text{EGH} &= 64^\circ \end{aligned}$$
(e)
$$\begin{aligned} \tan\theta &= \frac{\text{EH}}{\text{GB}} \\ &= \frac{6.2}{4} \\ \theta &= 57^\circ \end{aligned}$$

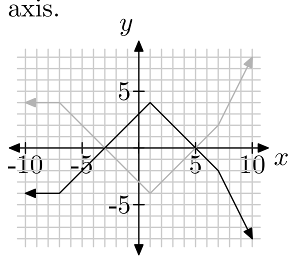
9.
$$\begin{aligned} \frac{\theta}{360} &= \frac{440}{2\pi \times 6350 \cos 37^\circ} \\ \theta &= \frac{440}{2\pi \times 6350 \cos 37^\circ} \times 360 \\ &= 5^\circ \\ \text{longitude} &= 126 + 5 \\ &= 131^\circ\text{E} \\ \frac{\alpha}{360} &= \frac{330}{2\pi \times 6350} \\ \alpha &= \frac{330}{2\pi \times 6350} \times 360 \\ &= 3^\circ \\ \text{latitude} &= 37 - 3 \\ &= 34^\circ\text{S} \end{aligned}$$
New position: 34°S, 131°W

10. For the triangle to have an obtuse angle, the longest side must be longer than the hypotenuse if it were right-angled, i.e. $c^2 > a^2 + b^2$. This yields two possibilities.

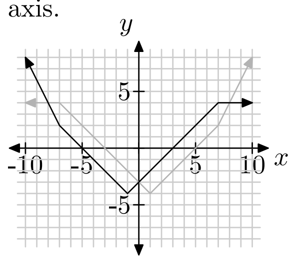
If x is the longest side, then
$$\begin{aligned} x^2 &> 5^2 + 9^2 \\ x &> \sqrt{106} \end{aligned}$$
Since it must also satisfy the triangle inequality x must be less than the sum of the other two sides. The solution in this case is $\sqrt{106} < x < 14$.

If x is not the longest side, then
$$\begin{aligned} 9^2 &> 5^2 + x^2 \\ x &< \sqrt{56} \\ x &< 2\sqrt{14} \end{aligned}$$
Since it must also satisfy the triangle inequality x must be greater than the difference between the other two sides. The solution in this case is $4 < x < 2\sqrt{14}$.

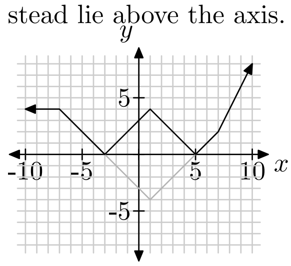
11. (a) $y = -\text{f}(x)$ represents a reflection in the x-axis.



(b) $y = \text{f}(-x)$ represents a reflection in the y-axis.



(c) $y = |\text{f}(x)|$ signifies that any part of $\text{f}(x)$ that falls below the x-axis will be reflected to instead lie above the axis.



(d) $y = \text{f}(|x|)$ signifies that any part of $\text{f}(x)$ that falls left of the y-axis will be discarded and replaced with a mirror image of the part of the function that lies to the right of the axis.