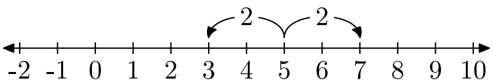
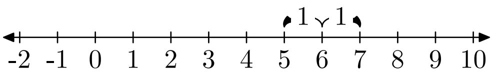
# Chapter 1

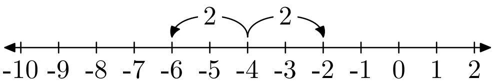
## Exercise 1A

1. 

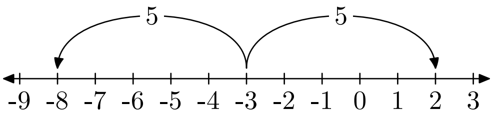
$x = 3$ or $x = 7$

2. 

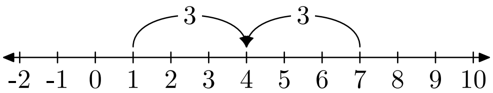
$x = 5$ or $x = 7$

3. 

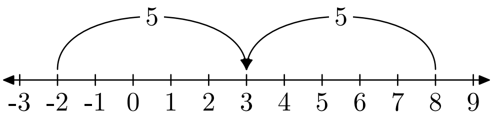
$x = -6$ or $x = -2$

4. 

$x = -8$ or $x = 2$

5. 

$x = 4$

6. 

$x = 3$

7. $x + 3 = 7$ or $x + 3 = -7$
$x = 4$ $\quad$ $x = -10$

8. $x - 3 = 5$ or $x - 3 = -5$
$x = 8$ $\quad$ $x = -2$

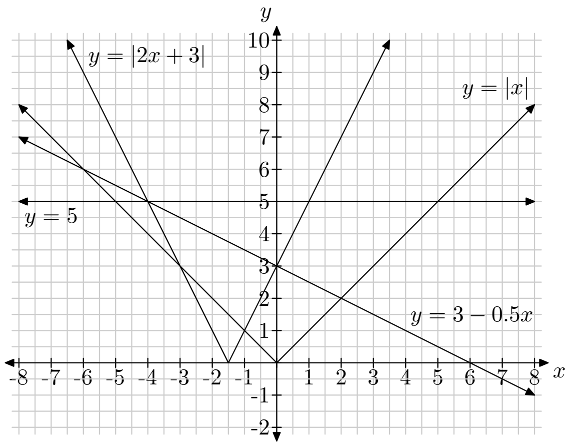
9. No solution (absolute value can never be negative).

10. $x - 2 = 11$ or $x - 2 = -11$
$x = 13$ $\quad$ $x = -9$

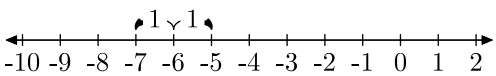
11. $2x + 3 = 7$ or $2x + 3 = -7$
$2x = 4$ $\quad$ $2x = -10$
$x = 2$ $\quad$ $x = -5$

12. $5x - 8 = 7$ or $5x - 8 = -7$
$5x = 15$ $\quad$ $5x = 1$
$x = 3$ $\quad$ $x = \frac{1}{5}$

13. Find the appropriate intersection and read the $x$-coordinate.
    (a) Intersections at (3,4) and (7,4) so $x = 3$ or $x = 7$.
    (b) Intersections at (-2,4) and (6,4) so $x = -2$ or $x = 6$.
    (c) Intersections at (4,2) and (8,6) so $x = 4$ or $x = 8$.

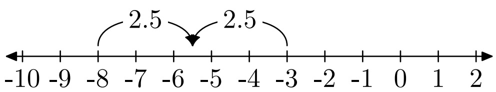
14. 

    (a) Intersections at (-4,5) and (1,5) so $x = -4$ or $x = 1$.
    (b) Intersections at (-6,6) and (2,2) so $x = -6$ or $x = 2$.
    (c) Intersections at (-4,5) and (0,3) so $x = -4$ or $x = 0$.
    (d) Intersections at (-3,3) and (-1,1) so $x = -3$ or $x = -1$.

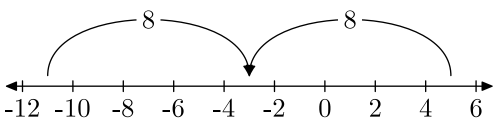
15. 

$x = -7$ or $x = -5$

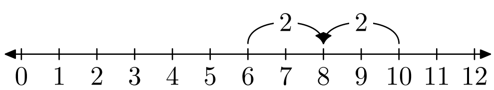
16. No solution (absolute value can never be negative).

17. 

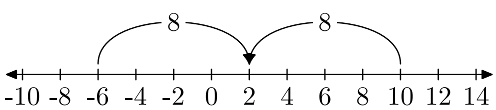
$x = -5.5$

18. 

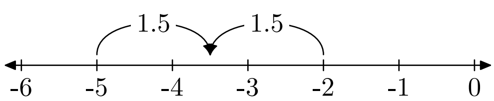
$x = -3$

19. 

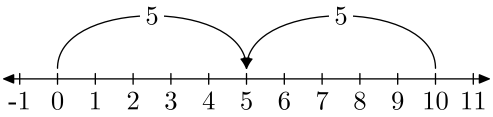
$x = 8$

20. 

$x = 2$

21. 

$x = -3.5$

22. 


$x = 5$

23.
$$\begin{aligned} x + 5 &= 2x - 14 \\ x &= 19 \end{aligned}$$

$$\begin{aligned} |19 + 5| &= |2 \times 19 - 14| \\ |24| &= |24| \quad \checkmark \end{aligned}$$

or

$$\begin{aligned} x + 5 &= -(2x - 14) \\ x + 5 &= -2x + 14 \\ 3x &= 9 \\ x &= 3 \end{aligned}$$

$$\begin{aligned} |3 + 5| &= |2 \times 3 - 14| \\ |8| &= |-8| \quad \checkmark \end{aligned}$$

24.
$$\begin{aligned} 3x - 1 &= x + 9 \\ 2x &= 10 \\ x &= 5 \end{aligned}$$

$$\begin{aligned} |3 \times 5 - 1| &= |5 + 9| \\ |14| &= |14| \quad \checkmark \end{aligned}$$

or

$$\begin{aligned} -(3x - 1) &= x + 9 \\ -3x + 1 &= x + 9 \\ -4x &= 8 \\ x &= -2 \end{aligned}$$

$$\begin{aligned} |3 \times -2 - 1| &= |-2 + 9| \\ |-7| &= |7| \quad \checkmark \end{aligned}$$

25.
$$\begin{aligned} 4x - 3 &= 3x - 11 \\ x &= -8 \end{aligned}$$

$$\begin{aligned} |4 \times -8 - 3| &= |3 \times -8 - 11| \\ |-35| &= |-35| \quad \checkmark \end{aligned}$$

or

$$\begin{aligned} 4x - 3 &= -(3x - 11) \\ 4x - 3 &= -3x + 11 \\ 7x &= 14 \\ x &= 2 \end{aligned}$$

$$\begin{aligned} |4 \times 2 - 3| &= |3 \times 2 - 11| \\ |5| &= |-5| \quad \checkmark \end{aligned}$$

26.
$$\begin{aligned} 5x - 11 &= 5 - 3x \\ 8x &= 16 \\ x &= 2 \end{aligned}$$

$$\begin{aligned} |5 \times 2 - 11| &= |5 - 3 \times 2| \\ |-1| &= |-1| \quad \checkmark \end{aligned}$$

or

$$\begin{aligned} -(5x - 11) &= 5 - 3x \\ -5x + 11 &= 5 - 3x \\ 6 &= 2x \\ x &= 3 \end{aligned}$$

$$\begin{aligned} |5 \times 3 - 11| &= |5 - 3 \times 3| \\ |4| &= |-4| \quad \checkmark \end{aligned}$$

27.
$$\begin{aligned} x - 2 &= 2x - 6 \\ -x &= -4 \\ x &= 4 \end{aligned}$$

$$\begin{aligned} |4 - 2| &= 2 \times 4 - 6 \\ |2| &= 2 \quad \checkmark \end{aligned}$$

or

$$\begin{aligned} -(x - 2) &= 2x - 6 \\ -x + 2 &= 2x - 6 \\ -3x &= -8 \\ x &= \frac{8}{3} \end{aligned}$$

$$\begin{aligned} |\frac{8}{3} - 2| &= 2 \times \frac{8}{3} - 6 \\ |\frac{2}{3}| &\neq -\frac{2}{3} \end{aligned}$$

The second 'solution' is not valid. The only solution is $x = 4$.

28.
$$\begin{aligned} x - 3 &= 2x \\ x &= -3 \end{aligned}$$

$$\begin{aligned} |-3 - 3| &= 2 \times -3 \\ |-6| &\neq -6 \end{aligned}$$

or

$$\begin{aligned} -(x - 3) &= 2x \\ -x + 3 &= 2x \\ -3x &= -3 \\ x &= 1 \end{aligned}$$

$$\begin{aligned} |1 - 3| &= 2 \times 1 \\ |-2| &= 2 \quad \checkmark \end{aligned}$$

The first 'solution' is not valid. The only solution is $x = 1$.

29.
$$\begin{aligned} x - 2 &= 0.5x + 1 \\ 0.5x &= 3 \\ x &= 6 \end{aligned}$$

$$\begin{aligned} |6| - 2 &= 0.5 \times 6 + 1 \\ 4 &= 4 \quad \checkmark \end{aligned}$$

or

$$\begin{aligned}-x-2&=0.5x+1\\-1.5x&=3\\x&=-2\end{aligned}$$

$$\begin{aligned}|-2|-2&=0.5\times-2+1\\0&=0\quad\checkmark.\end{aligned}$$

30. $$\begin{aligned}x+2&=-3x+6\\4x&=4\\x&=1\end{aligned}$$

$$\begin{aligned}|1+2|&=-3\times1+6\\|3|&=3\quad\checkmark\end{aligned}$$

or

$$\begin{aligned}-(x+2)&=-3x+6\\-x-2&=-3x+6\\2x&=8\\x&=4\end{aligned}$$

$$\begin{aligned}|4+2|&=-3\times4+6\\|6|&\neq3-6\end{aligned}$$

The second solution is invalid. The only solution is $x=1$.

31. $x\geq1$:

$$\begin{aligned}x+5+x-1&=7\\2x+4&=7\\2x&=3\\x&=1.5\quad\checkmark\end{aligned}$$

$-5\leq x\leq1$:

$$\begin{aligned}x+5-(x-1)&=7\\x+5-x+1&=7\\6&\neq7\quad\implies\text{no sol'n}\end{aligned}$$

$x\leq-5$:

$$\begin{aligned}-(x+5)-(x-1)&=7\\-x-5-x+1&=7\\-2x-4&=7\\-2x&=11\\x&=-5.5\quad\checkmark\end{aligned}$$

32. $x\geq4$:

$$\begin{aligned}x+3+x-4&=2\\2x-1&=2\\2x&=3\\x&=1.5\\&\implies\text{no sol'n (out of domain)}\end{aligned}$$

$-3\leq x\leq4$:

$$\begin{aligned}x+3-(x-4)&=2\\x+3-x+4&=2\\7&\neq2\quad\implies\text{no sol'n}\end{aligned}$$

$x\leq-3$:

$$\begin{aligned}-(x+3)-(x-4)&=2\\-x-3-x+4&=2\\-2x+1&=2\\-2x&=1\\x&=-0.5\\&\implies\text{no sol'n (out of domain)}\end{aligned}$$

The equation has no solution.

33. $x\geq3$:

$$\begin{aligned}x+5+x-3&=8\\2x+2&=8\\2x&=6\\x&=3\quad\checkmark\end{aligned}$$

$-5\leq x\leq3$:

$$\begin{aligned}x+5-(x-3)&=8\\x+5-x+3&=8\\8&=8\end{aligned}$$

$\implies$ all of $-5\leq x\leq3$ is a solution.

$x\leq-5$:

$$\begin{aligned}-(x+5)-(x-3)&=8\\-x-5-x+3&=8\\-2x-2&=8\\-2x&=10\\x&=-5\end{aligned}$$

Solution is $-5\leq x\leq3$.

34. $x\geq8$:

$$\begin{aligned}x-8&=-(2-x)-6\\x-8&=-2+x-6\\-8&=-8\end{aligned}$$

$\implies$ all of $x\geq8$ is a solution.

$2\leq x\leq8$:

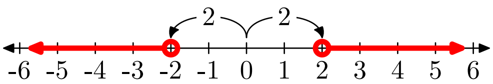
$$\begin{aligned}-(x-8)&=-(2-x)-6\\-x+8&=-2+x-6\\2x&=16\\x&=8\end{aligned}$$

$x\leq2$:

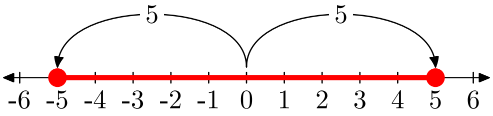
$$\begin{aligned}-(x-8)&=2-x-6\\-x+8&=-x-4\\8&\neq-4\quad\implies\text{no sol'n}\end{aligned}$$

Solution is $x\geq8$.

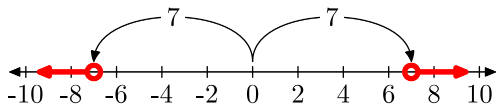
## Exercise 1B

1.


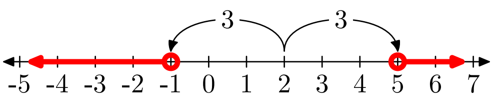

$-2 < x < 2$

2.


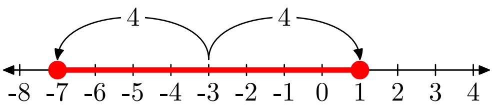

$-5 \leq x \leq 5$

3.


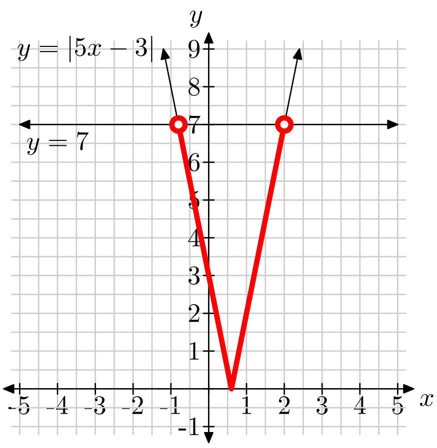

$x < -7$ or $x > 7$

4.



$x < -1$ or $x > 5$

5.



$-7 \leq x \leq 1$

6.



Algebraically:

For $5x - 3 \geq 0$:

$5x - 3 < 7$

$5x < 10$

$x < 5$

For $5x - 3 \leq 0$:

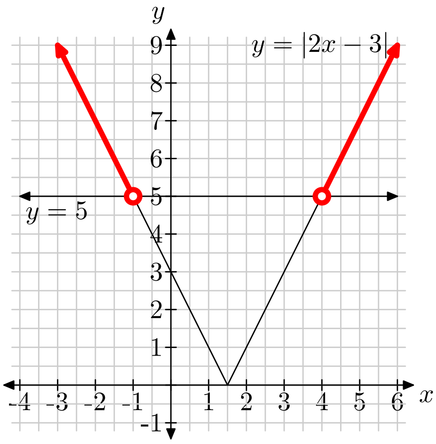
$-(5x - 3) < 7$

$5x - 3 > -7$

$5x > -4$

$x > -\frac{4}{5}$

$-\frac{4}{5} < x < 2$

7.



Algebraically:

For $2x - 3 \geq 0$:

$2x - 3 > 5$

$2x > 8$

$x > 4$

For $2x - 3 \leq 0$:

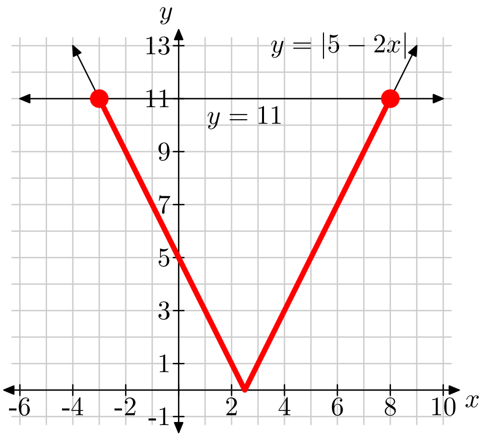
$-(2x - 3) > 5$

$2x - 3 < -5$

$2x < -2$

$x < -1$

$x < -1$ or $x > 4$

8.



Algebraically:

For $5 - 2x \geq 0$:

$5 - 2x \leq 11$

$-2x \leq 6$

$x \geq -3$

For $5 - 2x \leq 0$:

$-(5 - 2x) \leq 11$

$-5 + 2x \leq 11$

$2x \leq 16$

$x \leq 8$

$-3 \leq x \leq 8$

9. Centred on 0, no more than 3 units from centre: $|x| \leq 3$
10. Centred on 0, less than 4 units from centre: $|x| < 4$
11. Centred on 0, at least 1 unit from centre: $|x| \geq 1$
12. Centred on 0, more than 2 units from centre: $|x| > 2$
13. Centred on 0, no more than 4 units from centre: $|x| \leq 4$
14. Centred on 0, at least 3 units from centre: $|x| \geq 3$

15. Distance from 3 is greater than distance from 7. Distance is equal at $x = 5$ so possible values are $\{x \in \mathbb{R} : x > 5\}$.

16. Distance from 1 is less than or equal to distance from 8. Distance is equal at $x = 4.5$ so possible values are $\{x \in \mathbb{R} : x \leq 4.5\}$.

17. Distance from $-2$ is less than distance from 2. Distance is equal at $x = 0$ so possible values are $\{x \in \mathbb{R} : x < 0\}$.

18. Distance from 5 is greater than or equal to distance from $-1$. Distance is equal at $x = 2$ so possible values are $\{x \in \mathbb{R} : x \leq 2\}$.

19. Distance from 13 is greater than distance from 5. (Note $|5 - x| = |x - 5|$.) Distance is equal at $x = 9$ so possible values are $\{x \in \mathbb{R} : x < 9\}$.

20. Distance from $-12$ is greater than or equal to distance from 2. Distance is equal at $x = -5$ so possible values are $\{x \in \mathbb{R} : x \geq -5\}$.

21. Centred on 2, no more than 3 units from centre: $|x - 2| \leq 3$

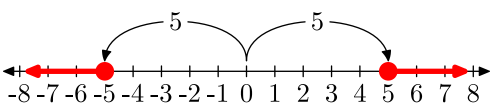
22. Centred on 3, less than 1 unit from centre: $|x - 3| < 1$

23. Centred on 2, at least 2 units from centre: $|x - 2| \geq 2$

24. Centred on 1, more than 2 units from centre: $|x - 1| > 2$

25. Centred on 1, no more than 4 units from centre: $|x - 1| \leq 4$

26. Centred on 1, at least 4 units from centre: $|x - 1| \geq 4$

27. 


$$x \leq -5 \text{ or } x \geq 5$$

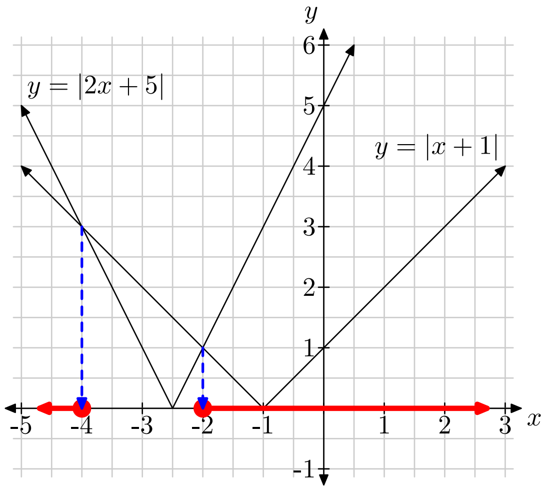
28. For $2x > 0$:
$2x < 8$
$x < 4$

For $2x < 0$:
$-2x < 8$
$2x > -8$
$x > -4$

$$-4 < x < 4$$

29. $|x| > -3$ is true for all $x$ (since the absolute value is always positive).

30. Distance from 3 is greater than or equal to distance from $-5$. Distance is equal at $-1$ so $x \leq -1$.

31. 


Algebraically:
First solve $|x + 1| = |2x + 5|$

$$\begin{aligned} x + 1 &= 2x + 5 \\ x &= -4 \end{aligned} \qquad \text{or} \qquad \begin{aligned} -(x + 1) &= 2x + 5 \\ -x - 1 &= 2x + 5 \\ -6 &= 3x \\ x &= -2 \end{aligned}$$

Now consider the three intervals delimited by these two solutions.

- $x < -4$
  Try a value, say -5:
  Is it true that $|-5 + 1| \leq |2(-5) + 5|$ ?
  Yes ($4 \leq 5$).
- $-4 < x < -2$
  Try a value, say -3:
  Is it true that $|-3 + 1| \leq |2(-3) + 5|$ ?
  No ($2 \not\leq 1$).
- $x > -2$
  Try a value, say 0:
  Is it true that $|0 + 1| \leq |2(0) + 5|$ ?
  Yes ($1 \leq 5$).

Solution set is

$$\{x \in \mathbb{R} : x \leq -4\} \cup \{x \in \mathbb{R} : x \geq -2\}$$

32. No solution (absolute value can not be negative.)

33. First solve $|5x + 1| = |3x + 9|$

$$\begin{aligned} 5x + 1 &= 3x + 9 \\ 2x &= 8 \\ x &= 4 \end{aligned} \qquad \text{or} \qquad \begin{aligned} -(5x + 1) &= 3x + 9 \\ -5x - 1 &= 3x + 9 \\ -10 &= 8x \\ x &= -1.25 \end{aligned}$$

Now consider the three intervals delimited by these two solutions.

- $x < -1.25$
  Try a value, say -2:
  Is it true that $|5(-2) + 1| > |3(-2) + 9|$ ?
  Yes ($9 > 3$).

- $-1.25 < x < 4$
  Try a value, say 0:
  Is it true that $|5(0) + 1| > |3(0) + 9|$ ?
  No ($1 \not> 9$).
- $x > 4$
  Try a value, say 5:
  Is it true that $|5(5) + 1| > |3(5) + 9|$ ?
  Yes ($26 > 24$).

Solution set is

$$\{x \in \mathbb{R} : x < -1.25\} \cup \{x \in \mathbb{R} : x > 4\}$$

34. First solve $|2x + 5| = |3x - 1|$

$$\begin{aligned} 2x + 5 &= 3x - 1 & \text{or} \quad -(2x + 5) &= 3x - 1 \\ x &= 6 & -2x - 5 &= 3x - 1 \\ & & -4 &= 5x \\ & & x &= -0.8 \end{aligned}$$

Now consider the three intervals delimited by these two solutions.

- $x < -0.8$
  Try a value, say -2:
  Is it true that $|2(-2) + 5| \geq |3(-2) - 1|$ ?
  No ($1 \not\geq 7$).
- $-0.8 < x < 6$
  Try a value, say 0:
  Is it true that $|2(0) + 5| \geq |3(0) - 1|$ ?
  Yes ($5 \geq -1$).
- $x > 6$
  Try a value, say 7:
  Is it true that $|2(7) + 5| \geq |3(7) - 1|$ ?
  No ($19 \not\geq 20$).

Solution set is

$$\{x \in \mathbb{R} : -0.8 \leq x \leq 6\}$$

Actually we only need to test one of the three intervals. At each of the two initial solutions we have lines crossing so if the LHS<RHS on one side of the intersection it follows that LHS>RHS on the other side, and vice versa. We'll use this in the next questions.

35. First solve $|6x + 1| = |2x + 5|$

$$\begin{aligned} 6x + 1 &= 2x + 5 & \text{or} \quad -(6x + 1) &= 2x + 5 \\ 4x &= 4 & -6x - 1 &= 2x + 5 \\ x &= 1 & -8x &= 6 \\ & & x &= -0.75 \end{aligned}$$

Now test one of the three intervals delimited by these two solutions.

- $x < -0.75$
  Try a value, say -1:
  Is it true that $|6(-1) + 1| \leq |2(-1) + 5|$ ?
  No ($5 \not\leq 3$).

Solution set is

$$\{x \in \mathbb{R} : -0.75 \leq x \leq 1\}$$

36. First solve $|3x + 7| = |2x - 4|$

$$\begin{aligned} 3x + 7 &= 2x - 4 & \text{or} \quad -(3x + 7) &= 2x - 4 \\ x &= -11 & -3x - 7 &= 2x - 4 \\ & & -5x &= 3 \\ & & x &= -0.6 \end{aligned}$$

Now test one of the three intervals delimited by these two solutions.

- $x < -11$
  Try a value, say -12:
  Is it true that $|3(-12) + 7| > |2(-12) - 4|$ ?
  Yes ($29 > 28$).

Solution set is

$$\{x \in \mathbb{R} : x < -11\} \cup \{x \in \mathbb{R} : x > -0.6\}$$

37. This is true for all $x \in \mathbb{R}$ since the absolute value is never negative, and hence always greater than -5.

38. First solve $|x - 1| = |2x + 7|$

$$\begin{aligned} x - 1 &= 2x + 7 & \text{or} \quad -(x - 1) &= 2x + 7 \\ x &= -8 & -x + 1 &= 2x + 7 \\ & & -3x &= 6 \\ & & x &= -2 \end{aligned}$$

Now test one of the three intervals delimited by these two solutions.

- $x < -8$
  Try a value, say -10:
  Is it true that $|-10 - 1| \leq |2(-10) + 7|$ ?
  Yes ($11 \leq 13$).

Solution set is

$$\{x \in \mathbb{R} : x < -8\} \cup \{x \in \mathbb{R} : x > -2\}$$

39. Distance from 11 is greater than or equal to distance from $-5$. 3 is equidistant, so $x \leq 3$

40. First solve $|3x + 7| = |7 - 2x|$

$$\begin{aligned} 3x + 7 &= 7 - 2x & \text{or} \quad -(3x + 7) &= 7 - 2x \\ 5x &= 0 & -3x - 7 &= 7 - 2x \\ x &= 0 & -x &= 14 \\ & & x &= -14 \end{aligned}$$

Now test one of the three intervals delimited by these two solutions.

- $x < -14$
  Try a value, say -20:
  Is it true that $|3(-20) + 7| > |7 - 2(-20)|$ ?
  Yes ($53 > 47$).

Solution set is

$$\{x \in \mathbb{R} : x < -14\} \cup \{x \in \mathbb{R} : x > 0\}$$

41. No solution (LHS=RHS $\forall x \in \mathbb{R}$)

42. True for all $x$ (LHS=RHS $\forall x \in \mathbb{R}$)

43. We can rewrite this as $3|x+1| \leq |x+1|$ which can only be true at $x+1=0$, i.e. $x=-1$.

44. We can rewrite this as $2|x-3| < 5|x-3|$ which simplifies to $2 < 5$ for all $x \neq 3$, so the solution set is
$$\{x \in \mathbb{R} : x \neq 3\}$$

45. First solve $x = |2x-6|$

$$\begin{aligned} x &= 2x-6 \quad\text{or}\quad & x &= -(2x-6) \\ x &= 6 & x &= -2x+6 \\ & & 3x &= 6 \\ & & x &= 2 \end{aligned}$$

Now test one of the three intervals delimited by these two solutions.

- $x < 2$
Try a value, say 0:
Is it true that $0 > |2(0)-6|$ ?
No ($0 \not> 6$).

Solution set is

$$\{x \in \mathbb{R} : 2 < x < 6\}$$

46. First solve $|x-3| = 2x$

$$\begin{aligned} x-3 &= 2x \quad\text{or}\quad & -(x-3) &= 2x \\ x &= -3 & -x+3 &= 2x \\ & & 3x &= 3 \\ & & x &= 1 \end{aligned}$$

The first of these is not really a solution, because it was found based on the premise of $x-3$ being positive which is not true for $x=-3$. As a result we really have only one solution. (Graph it on your calculator if you're not sure of this.)

Now test one of the two intervals delimited by this solution.

- $x < 1$
Try a value, say 0:
Is it true that $|0-3| \leq 2(0)$ ?
No ($3 \not\leq 0$).

Solution set is

$$\{x \in \mathbb{R} : x \geq 1\}$$

47. First solve $2x-2 = |x|$

$$\begin{aligned} 2x-2 &= x \quad\text{or}\quad & 2x-2 &= -x \\ x &= 2 & 3x &= 2 \\ & & x &= \frac{2}{3} \end{aligned}$$

The second of these is not really a solution, because it was found based on the premise of $x$ being negative which is not true for $x = \frac{2}{3}$. As a result we really have only one solution.

Now test one of the two intervals delimited by this solution.

- $x < 2$
Try a value, say 0:
Is it true that $2(0)-2 < |0|)$ ?
Yes ($-2 < 0$).

Solution set is

$$\{x \in \mathbb{R} : x < 2\}$$

48. First solve $|x|+1 = 2x$. If you sketch the graph of LHS and RHS it should be clear that this will have one solution with positive $x$:

$$\begin{aligned} x+1 &= 2x \\ x &= 1 \end{aligned}$$

The LHS is clearly greater than the RHS for negative $x$ so we can conclude that the solution set is
$$\{x \in \mathbb{R} : x \leq 1\}$$

49. Apart from having a $>$ instead of $\geq$ this problem can be rearranged to be identical to the previous one, so it will have a corresponding solution set:

$$\{x \in \mathbb{R} : x < 1\}$$

50. First solve $|x+4| = x+2$

$$\begin{aligned} x+4 &= x+2 \quad\text{or}\quad & -(x+4) &= x+2 \\ &\text{No Solution} & -x-4 &= x+2 \\ & & -2x &= 6 \\ & & x &= -3 \end{aligned}$$

The second of these is not really a solution, because it was found based on the premise of $x+4$ being negative which is not true for $x=-3$. As a result we have no solution. Graphically, the graphs of the LHS and RHS never intersect, so the inequality is either always true or never true. Test a value to determine which:
Try a value, say 0:
Is it true that $|(0)+4| > 0+2$ ?
Yes ($4 > 2$).

Solution set is $\mathbb{R}$.

51. "*" must be $>$ because we are including all values of $x$ greater than some distance from the central point.

At the value $x = 3$ we must have

$$\begin{aligned} |2x+5| &= a \\ |2\times 3+5| &= a \\ a &= 11 \end{aligned}$$

Then at $x = b$

$$\begin{aligned} -(2b+5) &= 11 \\ -2b-5 &= 11 \\ -2b &= 16 \\ b &= -8 \end{aligned}$$

52. Since 3 is a member of the solution set, resulting in the LHS being zero, the smallest possible absolute value, the inequality must be either $<$ or $\leq$. Since we have a filled circle at the starting point we can conclude that "*" is $\leq$.

    Point $x = 5$ is equidistant between 3 and $a$ (i.e. $|x-3| = |x-a|$ at $x = 5$ so we may conclude that $a = 7$.

53. First solve $|2x+5| = |x+a|$

$$\begin{aligned} 2x+5 &= x+a \quad \text{or} \quad & -(2x+5) &= x+a \\ x &= a-5 & -2x-5 &= x+a \\ & & -3x &= a+5 \\ & & x &= -\frac{a+5}{3} \end{aligned}$$

This gives us either

- $a-5=-2$ and $-\frac{a+5}{3} = -4$; or
- $a-5=-4$ and $-\frac{a+5}{3} = -2$

Only the second of these works, and we have $a = 1$

The open endpoints exclude $\leq$ and $\geq$ and all that remains is to test a value between $-2$ and $-4$ to decide between $<$ and $>$.

$$\begin{array}{ccc} |2(-3)+5| & * & |(-3)+1| \\ 1 & * & 2 \end{array}$$

and we see that "*" is $<$.

54. (a) False. This equation only holds when $x$ and $y$ are either both positive or both negative. For example, consider $x = 1$, $y = -2$: $|x+y| = 1$ but $|x|+|y| = 3$.
    (b) False. This equation only holds when $x$ and $y$ are not both positive or both negative, and further when $|x| \geq |y|$. For example, if $x = 1$ and $y = 2$, $|x+y| = 3$ but $|x|-|y| = -1$.
    (c) False. For example, consider $x = 1$, $y = -2$: $|x+y| = 1$ but $|x|+|y| = 3$.
    (d) True for all real values of $x$ and $y$.

## Miscellaneous Exercise 1

1. $$\begin{aligned} \text{distance} &= \sqrt{(-3-2)^2+(7--5)^2} \\ &= \sqrt{25+144} \\ &= 13 \end{aligned}$$

2. (a) $$\begin{aligned} f(2) &= 5(2)-3 \\ &= 7 \end{aligned}$$
   (b) $$\begin{aligned} f(-5) &= 5(-5)-3 \\ &= -28 \end{aligned}$$
   (c) $$\begin{aligned} f(1.5) &= 5(1.5)-3 \\ &= 4.5 \end{aligned}$$
   (d) $f(p) = 5p-3$
   (e) $$\begin{aligned} f(q) &= -18 \\ 5q-3 &= -18 \\ 5q &= -15 \\ q &= -3 \end{aligned}$$

3. (a) $8 = 2^3$
   (b) $64 = 8^2 = (2^3)^2 = 2^6$
   (c) $2^3 \times 2^7 = 2^{3+7} = 2^{10}$
   (d) $2^5 \times 16 = 2^5 \times 2^4 = 2^9$
   (e) $2^{10} \div 2^7 = 2^{10-7} = 2^3$
   (f) $2^7 \div 8 = 2^7 \div 2^3 = 2^4$
   (g) $256 \times 64 = 2^8 \times 2^6 = 2^{14}$
   (h) $1 = 2^0$

4. (a) $$\begin{aligned} 5^6 \times 5^x &= 5^{10} \\ 5^{6+x} &= 5^{10} \\ 6+x &= 10 \\ x &= 4 \end{aligned}$$
   (b) $$\begin{aligned} 27 \times 3^x &= 3^7 \\ 3^3 \times 3^x &= 3^7 \\ 3^{3+x} &= 3^7 \\ 3+x &= 7 \\ x &= 4 \end{aligned}$$

(c) $1\,000\,000 = 10^x$
$$10^6 = 10^x$$
$$x = 6$$

(d) $12^9 \div 12^x = 144$
$$12^{9-x} = 12^2$$
$$9 - x = 2$$
$$x = 7$$

(e) $2^3 \times 8 \times 2^x = 2^{10}$
$$2^3 \times 2^3 \times 2^x = 2^{10}$$
$$2^{3+3+x} = 2^{10}$$
$$6 + x = 10$$
$$x = 4$$

(f) $0.1 = 10^x$
$$10^{-1} = 10^x$$
$$x = -1$$

5. (a) $-5 < x < 5$
   (b) True for all $x$ (An absolute value is always greater than any negative number.)
   (c) $-6 \le 2x \le 6$ so $-3 \le x \le 3$
   (d) No value of $x$ satisfies this since an absolute value cannot be less than zero.
   (e) True for points on the number line having a distance from 3 less than their distance from 9, i.e. points nearer 3 than 9. The midpoint of 3 and 9 is 6 so the values of $x$ that satisfy the inequality are $x < 6$.
   (f) True for points on the number line nearer -1 than 5. The midpoint is 2, so $x < 2$.

6. Refer to Sadler's solutions for the sketches. These comments briefly describe the operations that have been enacted to produce these sketches.
   (a) Vertical reflection in the $x$-axis
   (b) Horizontal reflection in the $y$-axis
   (c) That part of the curve lying below the $x$-axis is vertically reflected in the $x$-axis.
   (d) That part of the curve lying to the left of the $y$-axis is replaced with a mirror image of the part lying to the right of the axis.

7. Each function is of the form $y = |a(x-b)|$ where $a$ represents the gradient of the positive slope and $b$ where it meets the $x$-axis. (It may be necessary to expand brackets if comparing these answers with Sadler's.)
   (a) Gradient 1, $x$-intercept -3: $y = |x + 3|$
   (b) Gradient 1, $x$-intercept 3: $y = |x - 3|$
   (c) Gradient 3, $x$-intercept 2: $y = |3(x - 2)|$
   (d) Gradient 2, $x$-intercept -2: $y = |2(x + 2)|$

8. (a) $f(3) = 3(3) - 2 = 7$
   (b) $f(-3) = 3(-3) - 2 = -11$
   (c) $g(3) = f(|3|) = f(3) = 7$
   (d) $g(-3) = f(|-3|) = f(3) = 7$
   (e) $f(5) = 3(5) - 2 = 13$
   (f) $g(-5) = f(|-5|) = f(5) = 13$
   (g) The graph of $f(x)$ is a line with gradient 3 and $y$-intercept -2. The graph of $g(x)$ is identical to that of $f(x)$ for $x \ge 0$. For $x < 0$ the graph is a reflection in the $y$-axis of the graph for positive $x$.

9. (a) The line lies above the curve for $x$ between $b$ and $e$ (but not including the extremes): $b < x < e$.
   (b) As for the previous question, but including the extremes: $b \le x \le e$.
   (c) The line is below the $x$-axis for $x < a$.
   (d) The line is above or on the $x$-axis for $x \ge a$.
   (e) The quadratic is above or on the $x$-axis for $x \le c$ or $x \ge d$.
   (f) The quadratic is above the $x$-axis for $x \le b$ or $x \ge e$.

10. Because $a$ is positive the sign of $ax$ is the same as the sign of $x$ and hence $|ax| = a|x|$. Similarly $|bx| = b|x|$.
$$|bx| > |ax|$$
$$b|x| > a|x|$$
Because $|x|$ is positive we can divide both sides by $|x|$ without being concerned about the inequality changing direction. This is, of course, only valid for $x \neq 0$
$$.b > a$$
which is true for all $x$ so we can conclude that the original inequality is true for all $x \neq 0$.