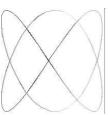# Investigation: Lissajous Figures

Jules Antoine Lissajous was a French physicist who lived from 1822 to 1880. Like many physicists of his time, Lissajous was interested in being able to see vibrations. He started off standing tuning forks in water and watching the ripple patterns, but his most famous experiments involved tuning forks and mirrors. For example, by attaching a mirror to a tuning fork and shining a light onto it, Lissajous was able to observe, via another couple of mirrors, the reflected light twisting and turning on the screen in time to the vibrations of the tuning fork. When he set up two tuning forks at right angles, with one vibrating at twice the frequency of the other, Lissajous found that the curved lines on the screen would combine to make a figure of eight pattern.

The ABC logo is a 3:1 Lissajous figure—if Lissajous wanted to see this pattern he would have to get one of his tuning forks to vibrate three times faster than the other.

Source: http://www.abc.net.au/science/holo/liss.htm

It is possible to simulate Lissajous' experiments mathematically using parametric equations. For example, plotting a graph with $x=\sin(2t)$, $y=\sin(t)$ generates Lissajous' figure 8.

The figures that can be generated like this are called **lissajous figures**.

Investigate Lissajous figures.

There are a number of ways you could do this. A couple of ideas a presented below, but you should follow your own ideas.

Some things you may consider exploring:

- What affect do changes in *amplitude* have?

- What do you get with different ratios of *x* and *y* period: try integer ratios (such as 2:1), simple non-integer ratios (such as 3:2) and other ratios (such as 2.1:1). Try to find a pattern so you can predict what the shape will look like for any given ratio.

- What is the effect of adding a *phase shift* to the *x* or *y* equation? For example how does Lissajous' figure 8 $x=\sin(2t)$, $y=\sin(t)$ change if we make it $x=\sin(2t)$, $y=\sin(t + \phi)$ for different values of ϕ?

Some ideas for investigation

1. Whatever else you do, it may be worthwhile plotting a few figures by hand on graph paper, to help get an understanding of how the figures are formed. Start with a table something like this:

t	**x=sin(2*t*)**	**y=sin(*t*)**

    and calculate *x* and *y* for enough values of *t* to complete the figure.

2. You could automate the manual calculation and plotting by using a spreadsheet. Your teacher can provide a sample spreadsheet if you don't want to make your own.

3. Use signal generators and an oscilloscope to generate lissajous figures electronically. See the separate sheet for details on how to set this up.

4. A number of useful resources can be found on the Internet. Try a search on "Lissajous Figures". For example, http://www.explorescience.com/activities/Activity_page.cfm?ActivityID=48 allows you to experiment with changing **amplitude**, **period** and **phase shift**. Another site also allows you to play with the colours of the plot: http://members.aol.com/edhobbs/applets/lissajous/

5. If you feel adventurous you could try to reproduce Lissajous' experiment with tuning forks, mirrors and lights, but don't expect this to be easy. We have a lot of things we can try that Lissajous did not have access to and we know a lot more about vibrations, partly because of his work. Try some of the other approaches first.

Generating lissajous figures electronically

If you can get access to the right equipment, you can play with lissajous figures without using computer simulations.

Up until the 1980s lissajous figures were used by radio and television engineers for comparing the frequency of transmitters and other equipment against a standard highly accurate signal source. Today other technologies are available that make this unnecessary, but we can still use the same equipment they used to explore lissajous figures.

You need:

- a cathode ray oscilloscope (sometimes called a CRO).
- two audio frequency signal generators
- Appropriate leads to connect the output of the signal generators to the inputs of the CRO.

First, connect only one signal generator to the input of the CRO. Set the signal generator to give a 1kHz sine wave output with an output level of about 1V. Set the CRO timebase to 100μs per division and input level to 0.1V per division. (These settings are not critical, they are just approximate starting points so don't worry if you can't work out how to get them right.) Adjust the settings on the CRO until you can see a sine wave pattern on the screen. Don't be afraid to play with the front panel controls on the CRO and signal generator to see what they do. The equipment is fairly robust, and short of dropping them you are not likely to do any damage.

When you have a sine wave nicely displayed on the screen, adjust the settings of the signal generator to see what effect changing the frequency and amplitude has on the display.

With this arrangement, the vertical movement of the beam on screen comes from the input from the signal generator. The horizontal movement of the beam across the screen comes from the internal time base in the CRO. Th is moves the beam from left to right at a constant speed. As a result we get a plot of y against t. For the next step we will replace the internal time base with the output from the second signal generator.

Turn the timebase control of the oscilloscope fully clockwise until it reads X. (If there is no X setting, there may be a switch or control on the back of the CRO that switches to the external timebase.) The beam should stop moving from left to right and you should just see a vertical line in the middle of the screen.

Now set the second signal generator to the same settings as the first and connect it to the second input on the CRO. Depending on the CRO, this may be on the front, or it may be a connection on the back. Now the horizontal movement of the beam is controlled by the second signal generator. Adjust the amplitude of the second signal generator so the beam stays on the screen. If you can't see anything on the screen then you probably need to reduce amplitude.

Now carefully adjust the frequency of one of the signal generators until you can see a lissajous figure. You should be able to generate many lissajous figures by selecting different frequency ratios. It will be easiest to 'tune in' the figures if you stick to fairly low frequencies (i.e. nothing higher than about 1kHz.) The trace is only stable if the two frquencies are *exactly* in the ratio of two integers. Because you are tuning it manually (and because the signal generators are not designed to give exact frequencies) your trace may seem to move a little no matter how hard you try.

Feel free to play with the equipment, but make notes and diagrams of what you do, and what settings you use, to include in your investigation report.